Half-maximal gauged supergravities from 10d heterotic DFT

Based on: F. Hassler, Y. Sakatani, LS, *Generalized Dualities for Heterotic and Type I Strings*, 2312.16283 [hep-th], submitted to JHEP.

Luca Scala

Wrocław University

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Uniwersytet Wrocławski

Generalised dualities

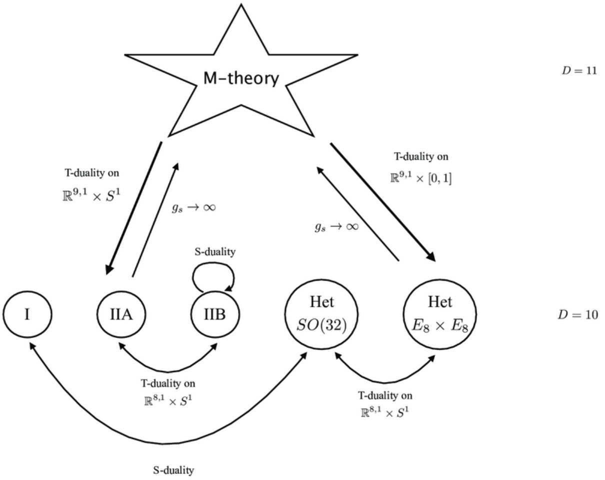

Figure – A. Fontanella, "Black Horizons and Integrability in String Theory", arXiv :1810.05434 [hep-th].

Supergravities

Superstring theories $\xrightarrow[\text{low energy limit}]{}$ SUGRAs.

Maximal [1] (32 supercharges)	Half-maximal (16 supercharges)
M-theory	Heterotic $E_8 \times E_8$
Type IIA	Heterotic SO(32)
Type IIB	Type I.

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$$\text{10-dim SUGRA} \underset{\text{uplifts}}{\overset{\text{consistent truncations}}{\rightleftarrows}} \text{(10-d)-dim SUGRA.}$$

Are there constraints on the procedure ?

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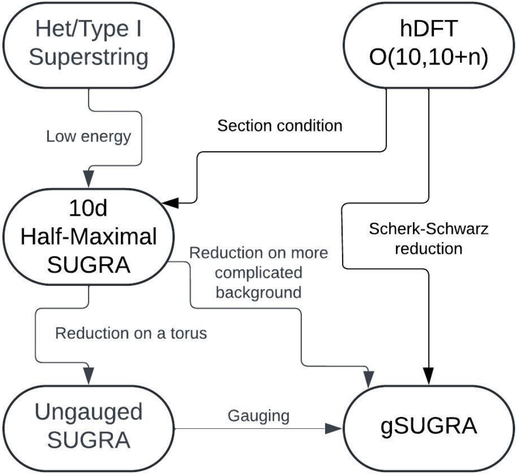
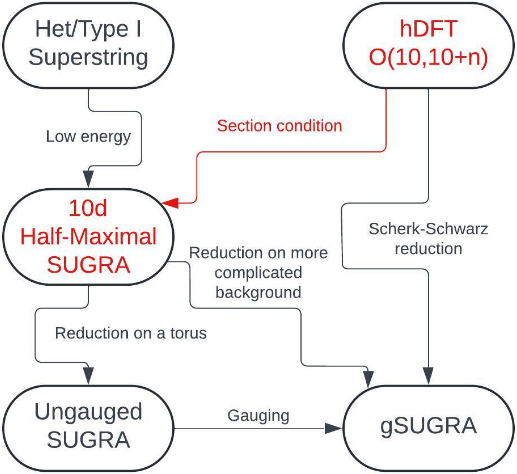
Dimensional reductions



From hDFT to SUGRA



Heterotic Double Field Theory (hDFT)

10-dimensional hDFT : $O(10, 10+n)$-covariant field theory defined by [2] :

$$\mathcal{L}_{\text{DFT}} = e^{-2d}\Big(-\tfrac{1}{12}\, H^{\mathbb{AD}}\, H^{\mathbb{BE}}\, H_{\mathbb{CF}}\, \hat{F}_{\mathbb{AB}}{}^{\mathbb{C}}\, \hat{F}_{\mathbb{DE}}{}^{\mathbb{F}} - \tfrac{1}{4}\, H^{\mathbb{AB}}\, \hat{F}_{\mathbb{AD}}{}^{\mathbb{C}}\, \hat{F}_{\mathbb{BC}}{}^{\mathbb{D}} + \\ + 2\, H^{\mathbb{AB}}\, D_{\mathbb{A}} F_{\mathbb{B}} + \tfrac{1}{6}\, \hat{F}_{\mathbb{AC}}{}^{\mathbb{D}}\, \hat{F}^{\mathbb{A}}{}_{\mathbb{D}}{}^{\mathbb{C}} - 2\, D_{\mathbb{A}} F^{\mathbb{A}} + F_{\mathbb{A}}\, F^{\mathbb{A}}\Big)\,, \tag{1}$$

2. O. Hohm, S. K. Kwak, *Double Field Theory Formulation of Heterotic Strings*, JHEP 06 (2011).

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where

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From 10-dim hDFT to 10-dim SUGRA (1)

To be well defined we have to impose the constraint

$$\partial^{\mathbb{I}} \cdot \partial_{\mathbb{I}} \cdot = 0. \tag{3}$$

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$$\mathrm{O}(10, 10+n) \rightarrow \mathrm{GL}(10) \times \mathcal{G}. \tag{4}$$

The constraint is solved by the canonical section

$$\partial_{\mathbb{I}} = \begin{pmatrix} \partial_m & \partial_{\mathcal{I}} & \partial^m \end{pmatrix} = \begin{pmatrix} \partial_m & 0 & 0 \end{pmatrix} . \tag{5}$$

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$$\Sigma_{\mathbb{I}\mathbb{J}}{}^{\mathbb{K}} = \begin{cases} f_{\mathcal{I}\mathcal{J}}{}^{\mathcal{K}} & \text{structure constants of } Lie(\mathcal{G}), \\ 0 & \text{otherwise.} \end{cases}$$

Standard parametrization

$$E_{\mathbb{A}}{}^{\mathbb{I}} = E_{\mathbb{A}}{}^{\mathbb{I}}(e, A, B_2), \qquad e^{-2d} := e^{-2\Phi}\sqrt{-g}\,, \tag{6}$$

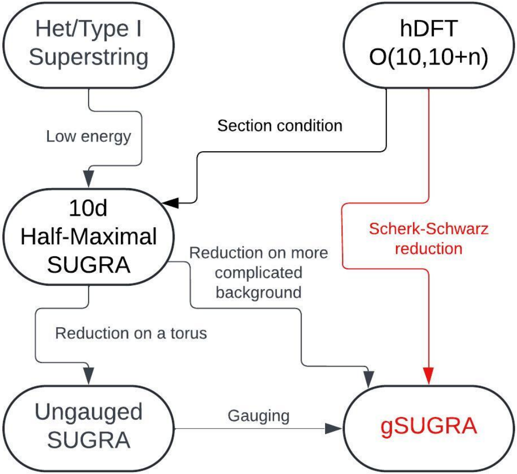
we find the 10-dim half-maximal SUGRA action

$$S = \int \mathrm{d}^{10}x\, \sqrt{-g}\, e^{-2\Phi}\big(R + 4\,\partial_m\Phi\,\partial^m\Phi - \tfrac{1}{12}\,\hat{H}_{mnp}\,\hat{H}^{mnp} - \tfrac{1}{4}\,\kappa_{\mathcal{I}\mathcal{J}}\,F_{mn}{}^{\mathcal{I}}\,F^{mn\mathcal{J}}\big)\,, \tag{7}$$

with field strengths

$$\hat{H}_3 := \mathrm{d}B_2 - \tfrac{1}{2}\,\kappa_{\mathcal{I}\mathcal{J}}\,A^{\mathcal{I}} \wedge \mathrm{d}A^{\mathcal{J}} - \tfrac{1}{3!}\,f_{\mathcal{I}\mathcal{J}\mathcal{K}}\,A^{\mathcal{I}} \wedge A^{\mathcal{J}} \wedge A^{\mathcal{K}}\,, \qquad F_2^{\mathcal{I}} := \mathrm{d}A^{\mathcal{I}} + \tfrac{1}{2}\,f_{\mathcal{J}\mathcal{K}}{}^{\mathcal{I}}\,A^{\mathcal{J}} \wedge A^{\mathcal{K}}\,. \tag{8}$$

Half-maximal gSUGRAs



Embedding tensor

Half-maximal gSUGRAs in $10 - d$ dimensions, $1 < d \leq 6$, are classified by the embedding of their gauge group G into the Lie group[3]

$$\mathrm{G}_D = \tilde{\mathrm{G}}_D(d) \times \mathrm{O}(d, d+n) \qquad \text{with} \qquad \tilde{\mathrm{G}}(d) = \begin{cases} \mathbb{R}^+ & 0 < d \leq 5 \\ \mathrm{SL}(2) & d = 6 \end{cases} . \tag{9}$$

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$${X_{\hat{A}\hat{B}}}^{\hat{C}}\xrightarrow[\text{IRREP decomp.}]{}\begin{cases}\{F_{ABC},\,\xi_A\} & d\leq 4\,,\\ \{F_{ABC},\,\xi_A,\,\boldsymbol{\vartheta}_A,\,\xi_{AB},\,\boldsymbol{\vartheta}_*\} & d=5\,,\\ \{F_{\alpha ABC},\,\xi_{\alpha A},\,\boldsymbol{\vartheta}_{\alpha A}\} & d=6\,.\end{cases} \tag{10}$$

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Geometric gaugings

From G_D frames

$$\mathcal{L}_{E_{\hat{A}}} E_{\hat{B}}{}^{\hat{I}} = -X_{\hat{A}\hat{B}}{}^{\hat{C}}\, E_{\hat{C}}{}^{\hat{I}}\,, \tag{11}$$

we want to obtain $\mathrm{O}(d, d+n)$ frames.

4. Y. Sakatani, *Half-maximal extended Drinfel'd algebras*, PTEP 2022 (2022) 1, 013B14.

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we want to obtain $\mathrm{O}(d, d+n)$ frames. We restrict indices

$$\mathcal{L}_{E_A} E_B{}^I = -X_{AB}{}^C\, E_C{}^I\,. \tag{12}$$

$d = 5$: we have to supplement to the section condition [4]

$$\partial_* = 0 \longrightarrow \xi_{AB} = \boldsymbol{\vartheta}_* = 0. \tag{13}$$

$d = 6$: we have to supplement to the section condition

$$\partial_{-I} = 0 \longrightarrow F_{-ABC} = 0. \tag{14}$$

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Truncating of the 10d action

Then Scherk-Schwarz reduction to truncate.

$$F_A = \mathrm{e}^{\Delta}\left(2\, D_A \phi - \xi_A - \partial_I E_A{}^I\right), \qquad F_{ABC} = X_{[ABC]}\,. \tag{15}$$

Computing the action it splits into

$$S = \int_{ext} \mathrm{d}^{10-d}x\, \mathcal{L}_{\mathrm{ext}} \int_{int} \mathrm{d}^d x\, v\,, \tag{16}$$

v : scalar density $\rightarrow$ left-invariant integration measure if the trombone gauging ($\boldsymbol{\vartheta}$'s) vanish.

We can, then, truncate the internal part.

Conclusions

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3 we found the frames corresponding to the dimensional-reduced gSUGRAs ;
4 we applied a Scherk-Schwarz reduction to obtain lower dimensional gSUGRAs from 10d hDFT.
5 (In the paper : generalised dualities coming from G$/H$ construction, explicit examples, constructing frames from random gaugings and an explicit example of a generalised duality. Analysed obstructions coming from the trombone gauging.)

Thanks for your attention !

Bibliography :

1. F. Hassler, Y. Sakatani, LS, *Generalized Dualities for Heterotic and Type I Strings*, 2312.16283 [hep-th], submitted to JHEP.
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5. J. Schon, M. Weidner, *Gauged N=4 supergravities*, JHEP 05 (2006).
6. E. Malek, *Half-Maximal Supersymmetry from Exceptional Field Theory*, Fortsch.Phys. 65 (2017) 10-11, 1700061.
7. Y. Sakatani, *Half-maximal extended Drinfel'd algebras*, PTEP 2022 (2022) 1, 013B14.

Preparing the lower dimensional reduction

We want to reduce on an internal d-dimensional space. We perform the following splitting of indices [5]

$$x^{\mathbb{I}} = \begin{pmatrix} x^{\mu} & x^{I} & x_{\mu} \end{pmatrix}, \qquad \text{with} \qquad x^{I} := \begin{pmatrix} x^{i} & x^{\mathcal{I}} & x_{i} \end{pmatrix}. \tag{17}$$

The decomposition reads

$$\begin{aligned} \mathrm{O}(10, 10+n) &\to \mathrm{GL}(10-d) \times \mathrm{O}(d, d+n) \qquad (18)\\ &\to \mathrm{GL}(10-d) \times \mathrm{GL}(d) \times \mathcal{G}\,. \qquad (19) \end{aligned}$$

$$\mathbb{E}_{\mathbb{A}}{}^{\mathbb{I}} = \begin{pmatrix} e_{\mathbf{a}}^{\mu} & -e_{\mathbf{a}}^{\nu}\, A_{\nu}{}^{I} & -e_{\mathbf{a}}^{\nu}\left(B_{\nu\mu} + \frac{1}{2} A_{\nu}{}^{K} A_{\mu K}\right) \\ 0 & \mathcal{V}_{A}{}^{I} & \mathcal{V}_{A}{}^{K} A_{\mu K} \\ 0 & 0 & e_{\mu}^{\mathbf{a}} \end{pmatrix}, \tag{20}$$

$$\mathrm{e}^{-2d} = \mathrm{e}^{-2\phi}\sqrt{-\det(g_{\mu\nu})}\,.$$

5. O. Hohm, H. Samtleben, *Gauge theory of Kaluza-Klein and winding modes*, Phys.Rev.D 88 (2013), 085005.

Generalized dualities

We constructed the frames

$$E_{\hat{A}}{}^{\hat{I}} = M_{\hat{A}}{}^{\hat{B}}\, V_{\hat{B}}{}^{\hat{J}}\, N_{\hat{J}}{}^{\hat{I}} \tag{21}$$

where

$$(M^{-1})_{\hat{A}}{}^{\hat{C}}\, \mathrm{d}M_{\hat{C}}{}^{\hat{B}} = -v^{\hat{C}}\, X_{\hat{C}\hat{A}}{}^{\hat{B}}\,, \tag{22}$$

$$N_{\hat{I}}{}^{\hat{J}} := \left[\exp(-\tfrac{1}{2!}\,\mathfrak{b}_{ij}\,R^{ij})\ \exp(-\mathfrak{a}_k^{\mathcal{I}}\,R_{\mathcal{I}}^k)\right]_{\hat{I}}{}^{\hat{J}}\,. \tag{23}$$

From the embedding tensor we extract the $(10-d)$-dimensional gSUGRA gauge group G. This is the Lie group corresponding to a Lie subalgebra of the Leibniz algebra of gaugings $((T_{\hat{A}})_{\hat{B}}{}^{\hat{C}} = -X_{\hat{A}\hat{B}}{}^{\hat{C}})$

$$T_{\hat{A}} \circ T_{\hat{B}} = X_{\hat{A}\hat{B}}{}^{\hat{C}}\, T_{\hat{C}}\,. \tag{24}$$

We choose a subgroup H of G (usually maximally isotropic), and the frames live in G$/H$. Choosing different H's result in the same physics : dualities.

Parametrization of the frame

$$
\mathbb{E}_{\mathbb{A}}{}^{\mathbb{I}} := \begin{pmatrix} e_{\hat{a}}^{n} & 0 & 0 \\ 0 & \nu_{\mathcal{A}}^{\mathcal{N}} & 0 \\ 0 & 0 & e_{n}^{\hat{a}} \end{pmatrix} \begin{pmatrix} \delta_{n}^{p} & -A_{n}{}^{\mathcal{P}} & -\frac{1}{2} A_{n}{}^{\mathcal{Q}} A_{p\mathcal{Q}} \\ 0 & \delta_{\mathcal{N}}^{\mathcal{P}} & A_{p\mathcal{N}} \\ 0 & 0 & \delta_{p}^{n} \end{pmatrix} \begin{pmatrix} \delta_{p}^{m} & 0 & -B_{pm} \\ 0 & \delta_{\mathcal{P}}^{\mathcal{M}} & 0 \\ 0 & 0 & \delta_{m}^{p} \end{pmatrix},
$$

$$
\mathrm{e}^{-2d} := \mathrm{e}^{-2\Phi} \sqrt{-g}\,, \tag{25}
$$