CRITICAL AND NEAR-CRITICAL RELAXATION OF HOLOGRAPHIC SUPERFLUIDS

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Stringtheory.pl, Kraków 08.06.2024 Based on 2209.09251 [hep-th] ► What is discrete scale invariance (DSI)?



▶ Critical relaxation in Holographic Superconductors







What is discrete scale invariance (DSI)?

Discrete scale invariance (DSI) is the symmetry underlying the self-similarity of fractal structures w.r.t scalings by a preferred factor.

Fractal	Name	Hausdorff dimension,		
		Preferred scaling factor		
	Sierpinski triangle	$\frac{\log(3)}{\log(2)}, 2$		
	Sierpinski carpet	$\frac{\log(8)}{\log(3)}$, 3		

Scale invariance (SI):

 $\mathcal{O}_{SI}(x) = \mu(\lambda)\mathcal{O}_{SI}(\lambda x)$ for any $\lambda \in \mathbb{R}^+$ and some $\mu(\lambda) = \lambda^{\alpha}$.

Discrete scale invariance (DSI):

$$\mathcal{O}_{DSI}(x) = \mu(\lambda_0)\mathcal{O}_{DSI}(\lambda_0 x)$$

only for a specific scale $\lambda_0 \in \mathbb{R}^+$ and the related scales $\lambda_0^m, m \in \mathbb{Z}$. Solution:

$$\mathcal{O}_{DSI}(x) \propto x^{\alpha}, \ \ \alpha = -\frac{\log \mu}{\log \lambda_0} + i \frac{2\pi n}{\log \lambda_0}, \ \ n \in \mathbb{Z}.$$

For $n \neq 0$, we hence find complex critical exponents and, because of

$$\mathcal{O}_{DSI}(x) \propto x^{\alpha} = x^{\Re(\alpha)} \left(\cos\left[\Im(\alpha)\log(x)\right] + i\sin\left[\Im(\alpha)\log(x)\right] \right),$$

log-periodic oscillations.

Discrete Scale Invariance (DSI) plays a role in:

• Fractals, stock markets and earthquakes $\begin{bmatrix} Sornette \\ 1998 \end{bmatrix}$

▶ Black hole formation [^{Choptuik}₁₉₉₃], [^{Hirschmann and Eardley}]

• The Efimov effect $\begin{bmatrix} Hammer and Platter \\ 2011 \end{bmatrix}$

• Quantum Gravity $\begin{bmatrix} Calcagni \\ 2017 \end{bmatrix}$

▶ Cyclic RG flows, see e.g. $\begin{bmatrix} Wilson \\ 1971 \end{bmatrix}$, $\begin{bmatrix} Bulycheva \text{ and } Gorsky \\ 2014 \end{bmatrix}$, $\begin{bmatrix} Balasubramanian \\ 2013 \end{bmatrix}$ → See my \bigcirc talk yesterday!

► AdS/CMT models: [Liu et al.; Faulkner et al.], [Hartnoll et al.], [2011; 2011
[Erdmenger et al.], [Brattan et al.], [Ammon et al.]
[2017

The bottom-up Kondo model

- Bottom-up holographic model of the Kondo effect $\begin{bmatrix} Erdmenger et al. \\ 2013 \end{bmatrix}$.
- Superficially similar to a holographic superconductor in AdS₂: Charged scalar field Φ, gauge field a_m.
- Double-trace boundary conditions imposed on Φ:
 Asymptotic expansion includes vev. and Kondo coupling κ.
- At $\kappa = \kappa_c$, a second order phase transition happens where scalar field in bulk condenses.
- In $\begin{bmatrix} Erdmenger & et & al. \\ 2017 \end{bmatrix}$, we studied quenches in this system by making κ time dependent.

In $\begin{bmatrix} Erdmenger et al. \\ 2017 \end{bmatrix}$, we studied quenches in this system by making κ time dependent:



Log-periodic oscillations after exactly critical quench ${[{}^{\rm Erdmenger\ et\ al.}_{2017}]}.$ Bulk fields:

$$\phi_1(t,z) = t^{\upsilon_{I,\phi}} \cos(\upsilon_R \log(2\pi Tt)) \tilde{\phi}(z) + \mathcal{O}(t^x, x < \upsilon_{I,\phi} < 0),$$

$$\phi_2(t,z) = t^{\upsilon_{I,\phi}} \sin(-\upsilon_R \log(2\pi Tt)) \tilde{\phi}(z) + \mathcal{O}(t^x, x < \upsilon_{I,\phi} < 0),$$

$$a_t(t,z) = \frac{Q}{z} + \mu + \mathcal{O}(t^x, x < 0)$$

with $v_{I,\phi} \approx -1/2$, $v_{R,\phi} \approx 3/2$.

Origin of these values? - Need to understand non-linear effects!

Open questions:

► Can we derive the values for $v_{I,\phi}, v_{R,\phi}$? - Need to understand non-linear effects!

▶ How can we approach the non-linear equations of motion? Ansätze?

▶ Is this behaviour generic, or specific to the Kondo model?

Critical relaxation

 in

Holographic Superconductors

Bulk theory of a *Holographic Superconductor* [Hartnoll et al.] [Hartnoll et al.]:

$$S = \int d^{4}x \sqrt{-g} \left[R - 2\Lambda + \frac{1}{2\kappa^{2}} \left(-\frac{1}{4 q^{2}} F_{\mu\nu} F^{\mu\nu} - |\mathcal{D}\varphi|^{2} - m^{2} |\varphi|^{2} \right) \right]$$

▶ Cosmological constant $\Lambda < 0$; → asympt. AdS solutions:

$$ds^{2} = \frac{1}{u^{2}} \left[-f(u) dt^{2} - 2 dt du + dx^{2} + dy^{2} \right].$$

Asymptotic boundary at u = 0.

- \blacktriangleright Complex bulk scalar $\varphi.$ Near boundary: $\varphi = \Psi u^2 + \dots$
- ► Bulk U(1)-gauge field A_{μ} . Near boundary: $A_t = A_t + \rho u + \dots$ Phenomenology:
 - For $\rho < \rho_c$, AdS-RN black hole is stable ($\varphi = 0$).
 - Second order phase transition at $\rho = \rho_c$.
 - Scalar hair forms for $\rho > \rho_c, \varphi \neq 0$.

Quench: Start with equilibrium state $\rho > \rho_c \ (\Rightarrow \varphi \neq 0)$, then artificially switch it to $\rho = \rho_c$ suddenly $\begin{bmatrix} Flory et al. \\ 2022 \end{bmatrix}$.

Observation: Universally,

$$\phi(t) \approx \frac{4.07}{\sqrt{t+\delta t}}$$

$$\dot{\psi}(t) - (\mathcal{A}_t(t) - \rho_c) \approx \frac{0.93}{t+\delta t}$$

for $t \gg 1$ with $\Psi = \phi e^{i\psi}$.

Power law instead of QNM-like exponential falloff! $\dot{\psi} \sim 1/t$ means $\psi(t) \sim \log(t)$, i.e. log-periodic oscillations of complex phase similar to [Hirschmann and Eardley].



 $|\mathcal{A}_t(t) - \rho_c|$ (solid lines), $|\Psi| \equiv \phi(t)$ (dashed lines), and $|\dot{\psi}(t)|$ (dotted lines) for multiple exactly critical quenches.

Boundary model

Via AdS/CFT, this corresponds to the physics of a superconductor (or superfluid), where the dynamics of Ψ should be described by something like a *Ginzburg-Landau* (or Gross–Pitaevskii) equation $\begin{bmatrix} Tsuneto \ et \ al. \\ 1998 \end{bmatrix}$ (~ Model F $\begin{bmatrix} Hohenberg \ and \ Halperin \\ 1977 \end{bmatrix}$).

We propose the nonlinear phenomenological equation

$$\begin{bmatrix} \partial_t - iC_1 \left(\mathcal{A}_t(t) - \rho + C_5 |\Psi(t)|^2 \right) \end{bmatrix} \Psi(t)$$

$$\equiv -(C_2 + iC_3) \left[|\Psi(t)|^2 - C_4(\rho - \rho_c) \right] \Psi(t)$$

with parameters $C_1 = 1$ and

C_2	\approx	0.03018	C_3	\approx	0.09308
C_4	\approx	4.09192	C_5	\approx	0.14967

determined by fitting to static behaviour and exponential falloff (at $t \gg 1$) after near-critical quenches $\begin{bmatrix} Flory \text{ et al.} \\ 2022 \end{bmatrix}$.

Exactly critical solutions:

$$\phi(t) = \frac{1}{\sqrt{2C_2t + \frac{1}{\phi_0^2}}} \approx \frac{4.07}{t^{1/2}} + \dots$$

$$\begin{split} \dot{\psi} &- C_1(\mathcal{A}_t - \rho_c) \\ &= \frac{C_1 C_5 + C_3}{2C_2 t + \frac{1}{\phi_0^2}} \approx \frac{0.94}{t} + \dots \end{split}$$

Initial condition ϕ_0 fixes shift on the time axis $(\sim \delta t)!$



 $|\mathcal{A}_t(t) - \rho_c|$ (solid lines), $|\Psi| \equiv \phi(t)$ (dashed lines), and $|\dot{\psi}(t)|$ (dotted lines) for multiple exactly critical quenches.

Near-critical solutions

$$\phi(t) = \sqrt{\frac{C_4(\rho - \rho_c)}{1 - \left(1 - \frac{C_4(\rho - \rho_c)}{\phi_0^2}\right)e^{-2C_2C_4t(\rho - \rho_c)}}}$$

describe the system not just at late, but already at early and intermediate times.

For early times $t < \frac{1}{\rho - \rho_c}$, this exact solution is well approximated by the critical solution (~ power law falloff).



Numerical (blue) and analytical (orange) results for a near-critical quench. Top: $\phi(t)$, bottom: $\dot{\psi}(t) - C_1 \mathcal{A}_t(t)$.

Near-critical solutions

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Top: $\phi(t)$, bottom: $|\dot{\psi} - C_1(\mathcal{A}_t - \rho_c)|$ for varying values of ρ_{final} .

Summary

- Exactly at the critical point, holographic systems (Kondo: [^{Erdmenger et al.}], Superconductor: [^{Flory et al.}]) may relax after perturbations in a power-law manner.
- ▶ This power law relaxation exhibits discrete scale invariance because of the rotation of the complex phase.
- ► For the holographic superconductor, this behaviour can be reproduced by a phenomenological model with high precision.
- Even near-critical quenches show approximate power-law behaviour at intermediate time scales.

Thank you very much for your attention



Back-up slides...



The top-down Kondo model

Holographic top-down model [Erdmenger, Hoyos, O'Bannon, Wu: 1310.3271]:

Brane setup:

	0	1	2	3	4	5	6	7	8	9
N D3	x	x	x	x						
$N_7 \text{ D7}$	x	x			x	x	x	x	x	x
N_5 D5	x				x	x	x	x	x	

- ▶ D3/D7 strings: chiral fermions in 1+1 d → electrons ψ_L .
- ► D3/D5 strings: slave fermions in 0+1 d \rightarrow impurity spin $\vec{S} = \chi^{\dagger} \vec{T} \chi$.
- ▶ D5/D7 strings: tachyonic scalar → Formation of Kondo cloud:

$$\left< \mathcal{O} \right> \equiv \left< \psi_L^{\dagger} \chi \right> \neq 0$$

Kondo model: Field theory side

 Spin-spin interaction of electrons with a localised magnetic impurity. This may be another type of atom (i.e. Fe in Au), or, in the *Anderson model*, an electron bound in a quantum dot:



▶ Impact on resistivity at low temperatures.

 At low temperature, electrons form a bound state around impurity, the Kondo cloud.

• Can be mapped to a 1 + 1 dimensional system $\begin{bmatrix} Affleck and Ludwig \\ 1991 \end{bmatrix}$:

$$H = \frac{v}{2\pi} \psi_L^{\dagger} i \partial_x \psi_L + \frac{v}{2} \lambda_K \delta(x) \vec{S} \psi_L^{\dagger} \vec{\tau} \psi_L$$

v: Fermi velocity, λ_K : Kondo coupling.

The bottom-up Kondo model

Idea: Construct top-down model, and strip it from everything that seems non-essential $\begin{bmatrix} Erdmenger \ et \ al. \\ 2013 \end{bmatrix}$.

- ▶ Dual gravity model has 2 + 1 (bulk-) dimensions.
- Localised spin impurity is represented by co-dimension one hypersurface ("brane") extending from boundary into the bulk.
- Finite T is implemented by BTZ black hole background.
- Superficially similar to a holographic superconductor in AdS₂: Charged scalar field Φ , gauge field a_m .
- Double-trace boundary conditions imposed on Φ:
 Asymptotic expansion includes vev. and Kondo coupling κ.
- At $\kappa = \kappa_c$, a second order phase transition happens where Φ condenses.



 $S = S_{CS}[A] - \int d^3x \delta(x) \sqrt{-g} \left(\frac{1}{4} f^{mn} f_{mn} + \gamma^{mn} (\mathcal{D}_m \Phi)^{\dagger} \mathcal{D}_n \Phi + V(\Phi^{\dagger} \Phi)\right)$

References

- I. Affleck and A. W. W. Ludwig. Universal noninteger 'ground state degeneracy' in critical quantum systems. *Phys. Rev. Lett.*, 67:161-164, 1991. doi: 10.1103/PhysRevLett.67.161.
- M. Ammon, M. Baggioli, A. Jimenez-Alba, and S. Moeckel. A smeared quantum phase transition in disordered holography. 2018.
- K. Balasubramanian. Gravity duals of cyclic RG flows, with strings attached. 2013.
- D. K. Brattan, O. Ovdat, and E. Akkermans. Scale anomaly of a Lifshitz scalar: a universal quantum phase transition to discrete scale invariance. 2017.
- K. M. Bulycheva and A. S. Gorsky. Limit cycles in renormalization group dynamics. Phys. Usp., 57: 171-182, 2014. doi: 10.3367/UFNe.0184.201402g.0182. [Usp. Fiz. Nauk184,no.2,182(2014)].
- G. Calcagni. Complex dimensions and their observability. Phys. Rev., D96(4):046001, 2017. doi: 10.1103/PhysRevD.96.046001.
- M. W. Choptuik. Universality and scaling in gravitational collapse of a massless scalar field. Phys. Rev. Lett., 70:9-12, 1993. doi: 10.1103/PhysRevLett.70.9.
- J. Erdmenger, C. Hoyos, A. O'Bannon, and J. Wu. A Holographic Model of the Kondo Effect. JHEP, 1312:086, 2013. doi: 10.1007/JHEP12(2013)086.
- J. Erdmenger, M. Flory, M.-N. Newrzella, M. Strydom, and J. M. S. Wu. Quantum Quenches in a Holographic Kondo Model. JHEP, 04:045, 2017. doi: 10.1007/JHEP04(2017)045.
- T. Faulkner, H. Liu, J. McGreevy, and D. Vegh. Emergent quantum criticality, Fermi surfaces, and AdS(2). Phys. Rev., D83:125002, 2011. doi: 10.1103/PhysRevD.83.125002.
- M. Flory, S. Grieninger, and S. Morales-Tejera. Critical and near-critical relaxation of holographic superfluids. 9 2022.
- D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner. Renormalization group flows from holography supersymmetry and a c theorem. Adv. Theor. Math. Phys., 3:363-417, 1999.
- H.-W. Hammer and L. Platter. Efimov physics from a renormalization group perspective. *Phil. Trans. Roy. Soc. Lond.*, A369:2679, 2011. doi: 10.1098/rsta.2011.0001.
- S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz. Holographic Superconductors. JHEP, 12:015, 2008a. doi: 10.1088/1126-6708/2008/12/015.

- S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz. Building a Holographic Superconductor. Phys. Rev. Lett., 101:031601, 2008b. doi: 10.1103/PhysRevLett.101.031601.
- S. A. Hartnoll, D. M. Ramirez, and J. E. Santos. Thermal conductivity at a disordered quantum critical point. JHEP, 04:022, 2016. doi: 10.1007/JHEP04(2016)022.
- E. W. Hirschmann and D. M. Eardley. Universal scaling and echoing in the gravitational collapse of a complex scalar field. *Phys. Rev. D*, 51:4198-4207, Apr 1995. doi: 10.1103/PhysRevD.51.4198. URL https://link.aps.org/doi/10.1103/PhysRevD.51.4198.
- P. C. Hohenberg and B. I. Halperin. Theory of dynamic critical phenomena. Rev. Mod. Phys., 49: 435-479, Jul 1977. doi: 10.1103/RevModPhys.49.435.
- H. Liu, J. McGreevy, and D. Vegh. Non-Fermi liquids from holography. Phys. Rev., D83:065029, 2011. doi: 10.1103/PhysRevD.83.065029.
- D. Sornette. Discrete scale invariance and complex dimensions. Phys. Rept., 297:239-270, 1998. doi: 10.1016/S0370-1573(97)00076-8.
- T. Tsuneto, T. Tsuneto, M. Nakahara, and C. U. Press. Superconductivity and Superfluidity. Cambridge University Press, 1998. ISBN 9780521570732. URL https://books.google.de/books?id=sWft4g5EytcC.
- K. G. Wilson. The Renormalization Group and Strong Interactions. Phys. Rev., D3:1818, 1971. doi: 10.1103/PhysRevD.3.1818.