A perturbed 2D CFT quantum circuit model

based on WIP with J. Erdmenger and J. Kastikainen

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Motivation

A new candidate to probe the BH interior: Complexity

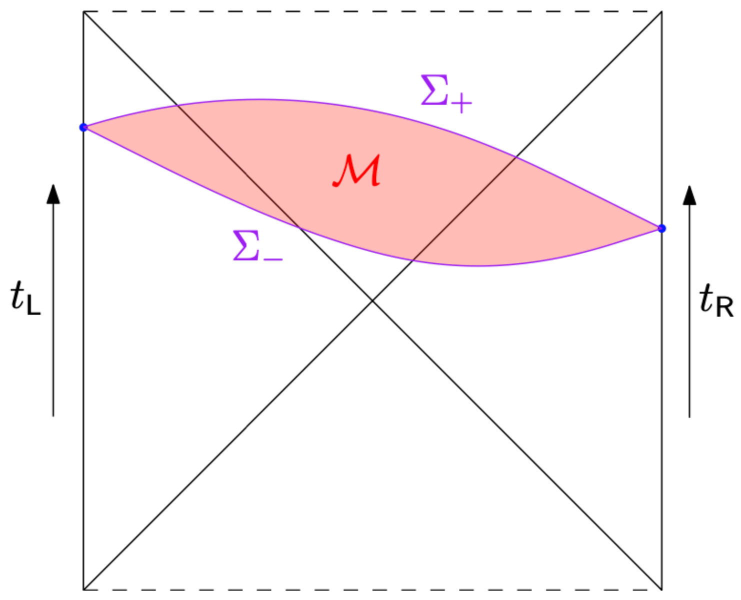
Conjecture: Duals of Complexity C probe the BH interior

Stanford, Susskind '14

QIS: Quantum Computational Complexity $C(U)$ of a unitary transformation U is given by the number of gates of the smallest circuit that implements U

Bdy: Nielsen circuit complexity? Krylov complexity? …?
Bulk: Volume? Action? Anything?

⇒ Need for quantitative match/mismatch analyses in holography



Belin, Myers, Ruan, Sarosi, Speranza '22

Motivation

Quantum circuit complexity in AdS_3/CFT_2

Focus of this talk on quantum circuits and quantum circuit complexity:

$$|\psi(\tau)\rangle = U(\tau)|\psi_r\rangle = \overleftarrow{\mathcal{T}} \exp\left(-i\int_{\tau_r}^{\tau} d\tau' H(\tau')\right)|\psi_r\rangle$$

$C(U(\tau)) = \min_{H(\tau)} \int_{\tau_r}^{\tau} d\tau' F(H(\tau'))$ F = 'cost function', $H(\tau)$ = 'generating Hamiltonian'

In AdS_3/CFT_2:

Idea: $H(\tau) = G_{f_\tau}$ = Generator of conformal transformation f_τ in 2d CFT Caputa, Magan '18

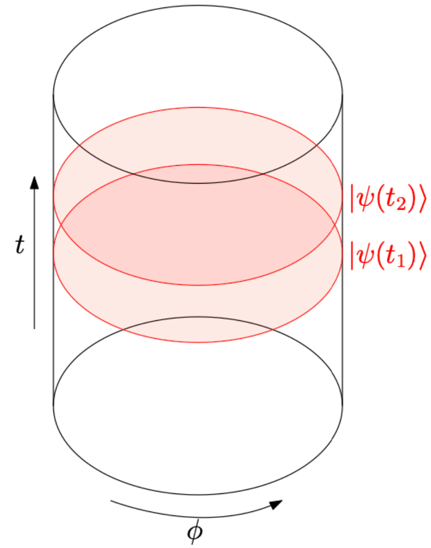
Motivation

Quantum circuit complexity in AdS_3/CFT_2

Progress in AdS_3/CFT_2: Two explicit **quantitative** dualities identified in context of complexity

1) Emergent spacetime from bdy circuit:

1)



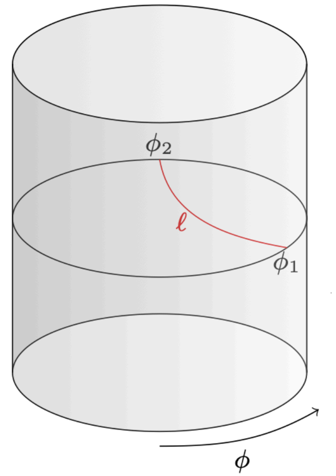
Bdy circuit states give rise to bulk geometry

$$H(t) = -\int_0^{2\pi} \sqrt{-g^{(bdy)}}\, T_t^{\ t} \overset{FG\, exp.}{\Rightarrow} g^{(bulk)}$$

Euclidean: Erdmenger, Flory, Gerbershagen, Heller, Weigel '21
Lorentzian: De Boer, Godet, Keski-Vakkuri, Kastikainen '23

2) Geometric dual of Fubini-Study cost function:

2)



$$\mathfrak{F}_{FS,bulk} = \int d\phi_1 \int d\phi_2 f(l)$$

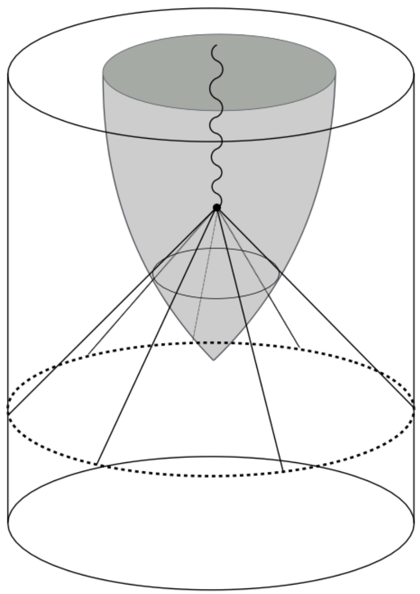
$$F_{FS,bdy} = \mathfrak{F}_{FS,bulk}$$

Bdy: Flory, Heller '20 Bulk: Erdmenger, Gerbershagen, Heller, Weigel '22

Goal of this talk

Extending AdS_3/CFT_2 ‘complexity testing ground’

Restriction of $H(t)$ to symmetry generator limits possible dual geometries to circuit



Inclusion of primary operator into $H(t)$ as another generator enables the circuit to change coadjoint orbit

Marginal perturbation hinted at to lead to Vaidya black hole collapse as dual geometry Anous, Hartman, Rovai, Sonner ’16

$\Rightarrow$ nontrivial (dynamic) testing grounds for complexity conjectures

Goal of this talk: Lay out the boundary construction, calculate cost function

Perturbed Conformal Circuit: The model

The starting point: Adding the perturbation

Previous circuit generator $Q(t) = G_{f_t,\bar{f}_t}$ Caputa, Magan '18

Let us construct a new generating Hamiltonian

$$H(t) = Q(t) + V(t)$$

$$V(t) = \int_0^{2\pi} d\phi\, J(t,\phi)\, F_t'(\phi)^{-h}\, \bar{F}_t'(\phi)^{-\bar{h}}\, \mathcal{O}_{h,\bar{h}}(\phi,\phi)$$

$\Rightarrow$

$|\psi_I(t)\rangle$: Expandable using Dyson series

$A_I(t)$: Standard conf. circuit operators

with $J(t,\phi) \ll 1$ $\Rightarrow$ Investigate first and second order perturbations

Perturbed Conformal Circuit: Results

Modifications can be obtained analytically

Example: $\langle Q(t) \rangle =$ unperturbed conf. circuit result + perturbation + $O(J^3)$

where perturbation is found to be

$$\int_{t_r}^{t} ds_2 \int_{t_r}^{s_2} ds_1 \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2\, J(s_2, \phi_2) J(s_1, \phi_1)$$

$$\left(u_t(x^-_{s_2}) \partial_{x^-_{s_2}} + h u_t'(x^-_{s_2}) + \bar{u}_t(x^+_{s_2}) \partial_{x^+_{s_2}} + \bar{h} \bar{u}_t'(x^+_{s_2}) \right)$$

$$\left(-i \langle 0 | \left[\mathcal{O}_{h,\bar{h}\, I}(x^-_{s_1}, x^+_{s_1}), \mathcal{O}_{h,\bar{h}\, I}(x^-_{s_2}, x^+_{s_2}) \right] | 0 \rangle \right)$$

$u_t(x^-) \equiv (\dot{F} \circ f)(t, x^-)$

$\bar{u}_t(x^+) \equiv (\dot{\bar{F}} \circ \bar{f})(t, x^-)$

With

$f, \bar{f}$

$h, \bar{h}$

$J(t, \phi)$

general

Perturbed Conformal Circuit: Results

The key integral

Key integral under assumptions $h = \bar{h} = 1$ and source $J(t, \phi) = J_t(t)\, J_\phi(\phi)$:

$$\mathcal{I}_I(t_1 - t_2) = \langle 0 | \mathcal{R}(t_1)\mathcal{R}(t_2) | 0 \rangle \quad \text{with} \quad \mathcal{R}(t) = \int_0^{2\pi} d\phi J_\phi(\phi)\mathcal{O}_I(t, \phi)$$

We can solve $\mathcal{I}(\Delta s)$ by Fourier expanding $J_\phi(\phi)$ and employing the properties of the modes of the primary two-pt. function to obtain

$$\mathcal{I}(\Delta t) = 2\pi^2 \sum_{n=0}^{\infty} |J_n|^2 e^{-i|n|\Delta t}(-|n| + i\cot\Delta t)\csc^2\Delta t$$

where we importantly notice that $\mathcal{I}(-\Delta t) = \mathcal{I}^*(\Delta t)$ and $Re\mathcal{I}(\Delta t \to 0)$ finite.

Perturbed Conformal Circuit: Cost

A result for the cost function modification in a simple case

For trivial time evolution $Q(t) = H_0 = L_0 \otimes 1 + 1 \otimes \bar{L}_0$ and step function time profile of the source $J_t(t) = J\,\Theta(t)$ we find*

$$F_{FS}(t) = Var\, H(t) = J^2 \begin{cases} 0\,, & t < 0 \\ 5\, Re\, \mathcal{I}(0) - 4\, Re\, \mathcal{I}(t)\,, & t > 0 \end{cases}$$

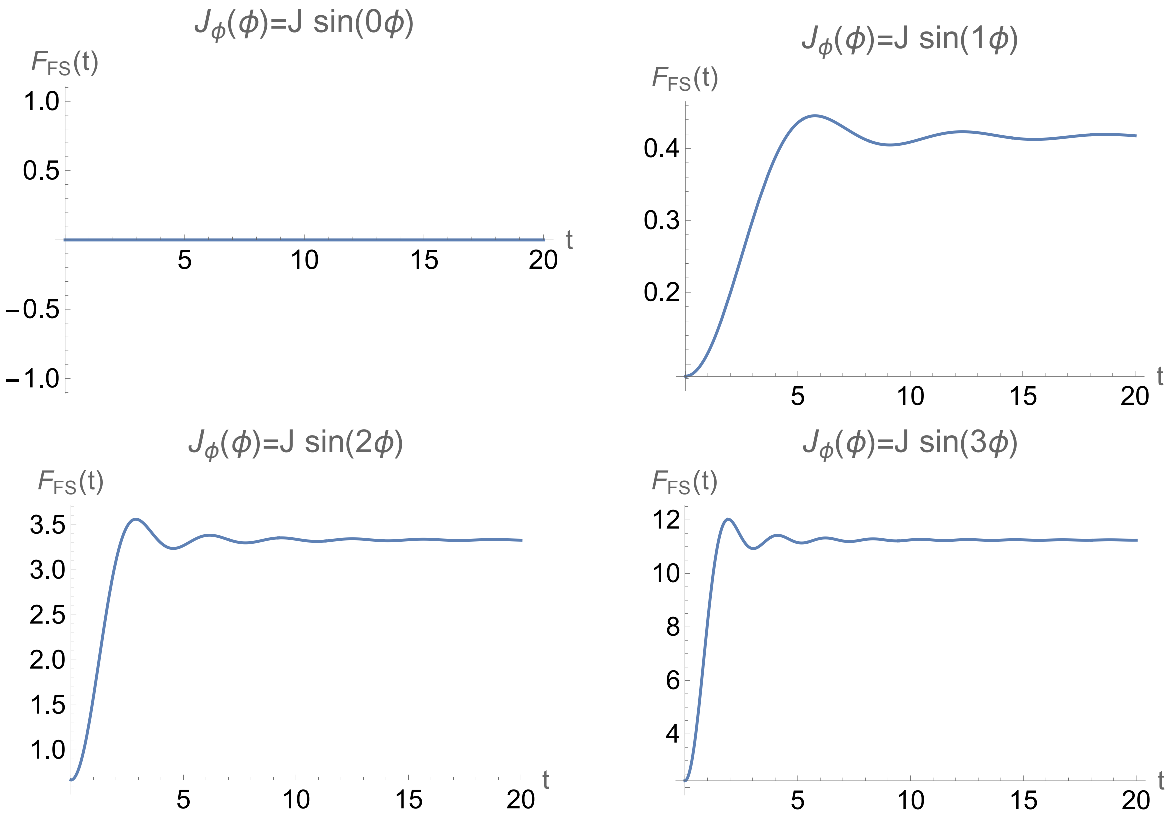
Notice that for the calculation we have worked in the geometry of the Lorentzian cylinder but we can take the Minkowski limit sending

$$t \to \frac{2\pi}{L} t\,, \quad n \to \frac{L}{2\pi} n\,, \quad L \to \infty \quad \text{in} \quad Re\mathcal{I}(t)$$

*one more argument about smearing of operators is necessary for $\mathcal{I}(0)$ contributions

Perturbed Conformal Circuit: Cost

Modified cost function: Minkowski Limit

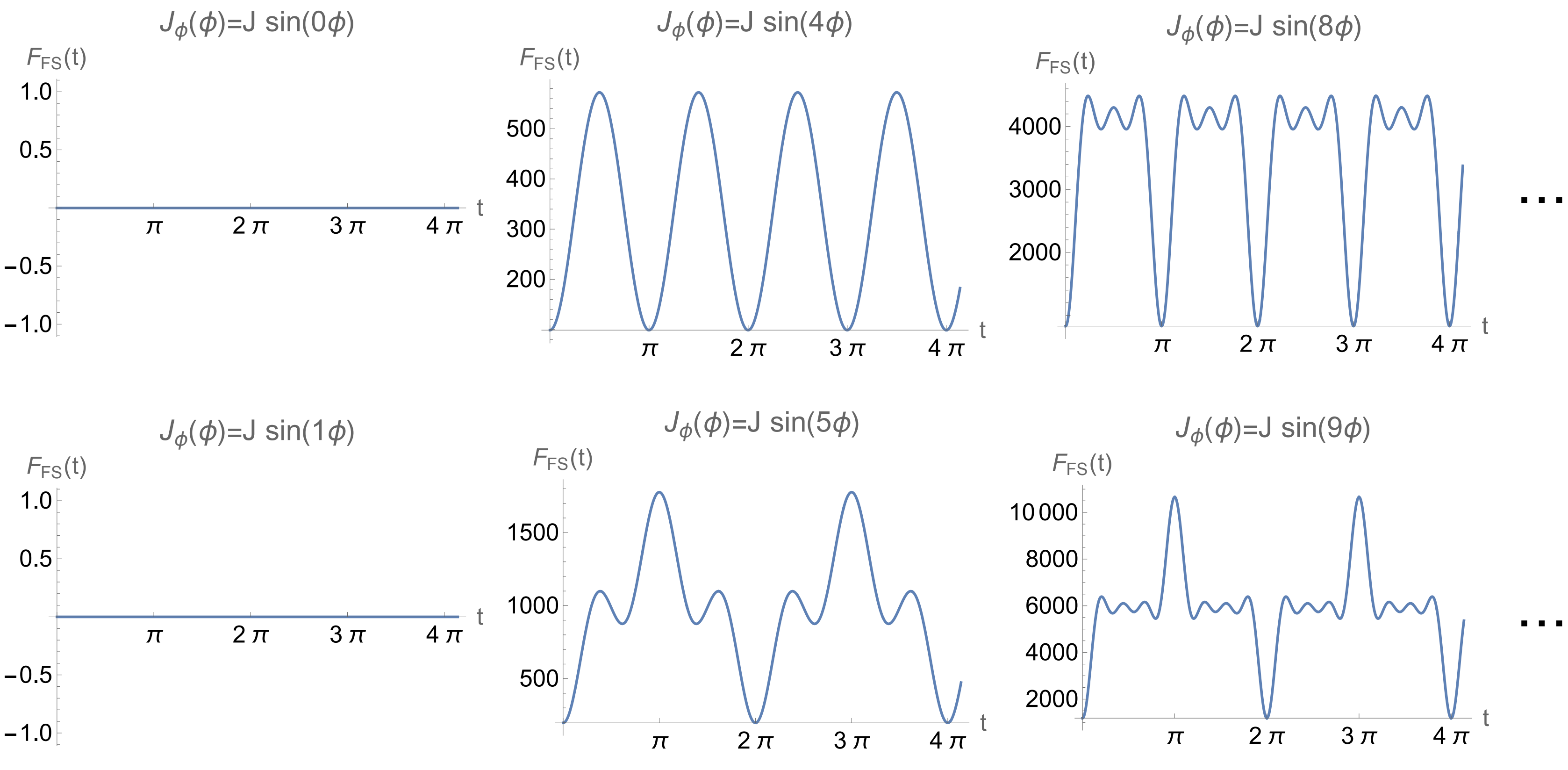


...

- No cost for global perturbation
- Equilibration towards $F_{FS,eq} = c_{Mink} \cdot n^3$ $\Rightarrow$ linear growth of integrated cost (complexity)
- Faster equilibration for higher mode sources

Perturbed Conformal Circuit: Cost

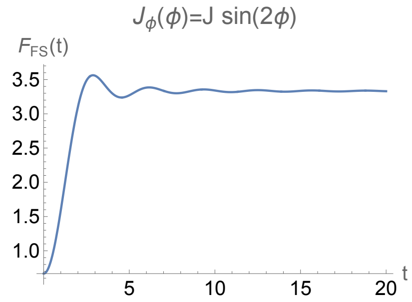
Modified cost function: Lorentzian Cylinder



- Equilibration within π intervals towards $F_{FS,eq} = c_{Cyl} \cdot n^3$
- In t, multiples of π are special points
- Slight difference between even and odd source modes

Summary of results

A perturbed conformal circuit on the boundary



- Constructed boundary framework for quantum circuit in CFT_2 that includes perturbation by a primary operator in generating Hamiltonian (can be viewed as quenching the CFT)

  ⇒ This enables circuit to switch coadjoint orbit

- For the Fubini-Study cost function, an (importantly!) finite result is obtained

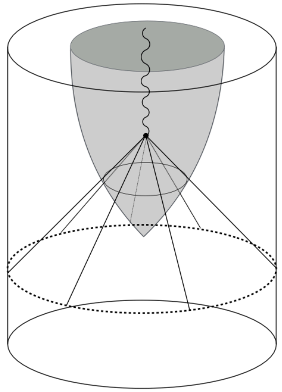
- Growth of integrated cost (hence the complexity upper boundary) is found to be linear at late times. Reason: Equilibration of the cost function found for all tested source modes.

  On Lorentzian cylinder, times that are multiples of π are special, presumably because signals from local excitations meet after circling the cylinder

Outlook

Gravity dual completes extension of ‘complexity testing ground’

- Gravity dual: Scaling of the energy expectation value $\langle H \rangle$ for circuit with nonzero source for interval of duration η hints at dual geometry: Vaidya black hole collapse with BH mass $M = J^2/\eta^2$ Anous, Hartman, Rovai, Sonner ’16 Das, Galante, Myers ‘14



$\Rightarrow$ open questions: backreaction? classification of cases where BH formes?

Bhattacharyya, Minwalla ‘09

- $\mathfrak{F}_{FS,bulk}$ naturally obtainable because perturbation based on primary operator 2pt function

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- Formulate other notions of complexity for (perturbed) conformal circuits (e.g. Krylov complexity of formation WIP Bhattacharya, Flory, Heller, Rizza, TS) $\Rightarrow$ Quantitative comparisons

Thank you!