# Quantum matter near a cosmological singularity

Alexandre Serantes, UGent

*Based on 2312.11643 with Jorge Casalderrey-Solana and David Mateos*

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# Motivation

- Spacetime singularities continue to be one of the most perplexing predictions of GR.
- Our current generic understanding comprises global and local approaches: singularity theorems, Belinski–Khalatnikov–Lifshitz (BKL) conjecture.
- This understanding is *essentially* classical: not only not quantum gravitational, but not even classical gravity + quantum matter effects (e.g. violation of energy conditions).
- In this talk I explore such quantum matter effects in the BKL context.

# BKL dynamics (I)

The BKL conjecture is a proposal for the behavior of spacetime geometry near a *generic* spacelike singularity [Belinski, Khalatnikov& Lifshitz; 1960s & 1970s]

**Main features**

- The dynamics are ultralocal.
- At each spatial point, the metric undergoes an infinite number of chaotic oscillations.

$$\ldots \to ds^2_{K_i} \to ds^2_{K_{i+1}} \to ds^2_{K_{i+2}} \to \ldots$$

Long Kasner epochs with fast transitions between them

# Interlude: the Kasner universe

The Kasner spacetime is a homogeneous but anisotropic vacuum solution of GR [Kasner; 1921]

$$ds^2 = -dt^2 + \sum_{i=1}^{d-1} (-t)^{2p_i} dx_i^2, \quad \sum_{i=1}^{d-1} p_i = \sum_{i=1}^{d-1} p_i^2 = 1.$$

In $d = 4$,

$$p_1 = \frac{1+u}{1+u+u^2}, \quad p_2 = \frac{u(1+u)}{1+u+u^2}, \quad p_3 = -\frac{u}{1+u+u^2},$$

$$p_1(1/u) = p_2(u),\ p_2(1/u) = p_1(u),\ p_3(1/u) = p_3(u),$$

$$u \geq 1.$$

For finite $u \geq 1$:

- Two directions contract & one direction expands.
- The spatial volume element contracts $\sqrt{-g^{(3)}} = |t|$: there is a spacelike curvature singularity at $t = 0$.
- $u = 1$, degenerate case: $p_1 = p_2 = \frac{2}{3}$, $p_3 = -\frac{1}{3}$; $u \to \infty$, Minkowski space in Milne coordinates.

# BKL dynamics (II)

In BKL dyamics:

- The transition between successive Kasner universes is driven by the expanding direction.
- In terms of Kasner exponents,

$$p_1(u) \to p_3(u-1), \quad p_2(u) \to p_2(u-1), \quad p_3(u) \to p_1(u-1),$$

  with the appropriate inversion if $u - 1 < 1$.
- The Kasner exponent transition rules are captured by a Bianchi type II vacuum solution.

# The purpose of this talk

The BKL dynamics have been shown to persist in the presence of a wide variety of forms of classical matter...

**What is the status of the BKL dynamics in the presence of quantum matter?**

- This is tough question: problem in nonequilibrium QFT in a curved spacetime with extremely violent dynamics.
- Ultimate goal: utilize holography to address this question for large $N$, strongly interacting QFTs.
- Long-term objective: requires solving self-consistently the four-dimensional boundary Einstein equations sourced by the holographic stress-energy tensor prescried by the five-dimensional bulk dynamics.

**In this talk I will report the first steps in this direction**

# Strategy

0. Start by restricting ourselves to the basic building block of the BKL dynamics: the Kasner universe.

1. Utilize holography to determine the dynamics of the quantum stress-energy tensor $\langle T_{\mu\nu} \rangle$.

2. Utilize boundary Einstein gravity to find the imprint of $\langle T_{\mu\nu} \rangle$ on the Kasner metric at leading order in the boundary gravitational coupling.

# Take-home messages

1. The Kasner metric inevitably drives the holographic QFT out-of-equilibrium.
2. $\langle T_{\mu\nu} \rangle$ behaves universally in the vicity of the spacelike singularity.
3. The leading-order backreaction of $\langle T_{\mu\nu} \rangle$ leads to a universal correction of the Kasner exponents suggesting that the BKL behavior might be avoided in semiclassical gravity.

# Holographic construction

**Action**:

- Five-dimensional Einstein gravity with negative cosmological constant $\Lambda = -6$ and a $m^2 = -3$ scalar field,

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{2}R - (\partial\phi)^2 - 2V(\phi)\right) + S_{GH} + S_{ct},$$

$$V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \dots$$

- We work in the standard quantization: $\phi$ is dual to a scalar relevant primary operator $\mathcal{O}$ of conformal dimension $\Delta = 3$.

**Boundary conditions**:

- The boundary metric is the Kasner spacetime.

- We break explicitly the conformal symmetry of the boundary CFT by turning on a constant source $M$ for $\mathcal{O}$,

$$S_{CFT} \to S_{QFT} \equiv S_{CFT} + M\int \mathcal{O}.$$

**States**:

- We focus on mixed QFT states that inherit the spatial translational invariance of the boundary Kasner metric.

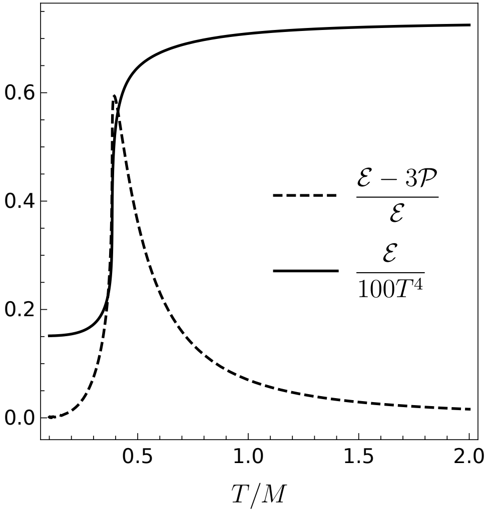
➥ There is an AH at the initial time slice.

➥ In EF coordinates, the bulk geometry is parameterized as

$$ds^2 = -Adt^2 + 2dtdr + \Sigma^2 \left( e^{B_1+\sqrt{3}B_2} dx_1^2 + e^{B_1-\sqrt{3}B_2} dx_2^2 + e^{-2B_1} dx_3^2 \right),$$

with $A$, $\Sigma$, $B_1$, $B_2$ and $\phi$ functions of $t$ and $r$ only.

For illustrative purposes, I choose a $V(\phi)$ that describes a RG flow between a UV and an IR fixed point.



## Message I: the holographic QFT is inevitably driven out-of-equilibrium

- By symmetry,

$$T^{\mu}_{\nu}(t) = \frac{N^2}{2\pi^2}\mathrm{diag}\left(-\mathcal{E}(t), \mathcal{P}_1(t), \mathcal{P}_2(t), \mathcal{P}_3(t)\right)$$

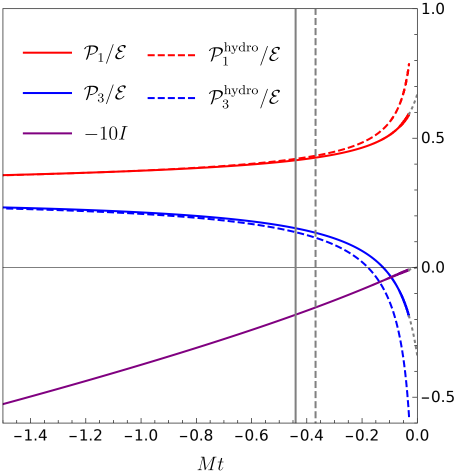
- **Diagnostic**: compare the microscopic pressures, $\mathcal{P}_i(t)$, with the pressures predicted by first-order relativistic hydrodynamics, $P_i^{\mathrm{hydro}}(t)$.

  In the Landau frame,

$$T_{\mu\nu} = \epsilon U_\mu U_\nu + P_{\mathrm{eq.}}(\epsilon)\Delta_{\mu\nu} - \eta(\epsilon)\sigma_{\mu\nu} - \xi(\epsilon)\Delta_{\mu\nu}(\nabla \cdot U) + \ldots$$

  $\epsilon$: energy density
  $U_\mu$: unit-normalized fluid velocity
  $\Delta_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$: projector
  $P_{\mathrm{eq.}}$: equilibrium EoS
  $\sigma_{\mu\nu}$: shear tensor (symmetric, transverse and traceless part of the velocity gradient)
  $\eta$: shear viscosity
  $\xi$: bulk viscosity

**Even if we start with a locally equilibrated state, the holographic QFT is driven eventually driven out-of-equilibrium as the Kasner singularity is approached**




$t_i = -20,\ p_1 = p_2 = \frac{2}{3},\ p_3 = -\frac{1}{3}$

# Message II: the near-singularity dynamics is universal

# Empirical finding

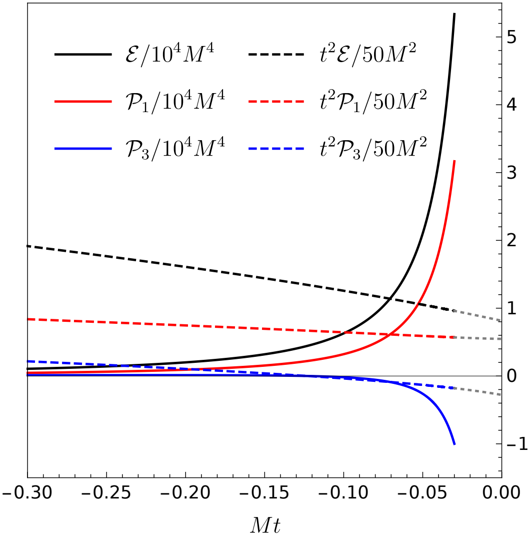
- In every example, $\langle T_{\mu\nu}\rangle$ features a power-law divergence as $t \to 0^-$ and the Kasner singularity is approached,

$$\langle T_{\mu\nu}\rangle \sim \frac{A_{\mu\nu}}{t^2}.$$

➥ Since $\mathcal{E}$ diverges, **the near-singularity behavior is solely controlled by the UV fixed point.** Compatible with the fact that $I \to 0$.

- In the $t \to 0^-$ limit, the numerical simulations are compatible with the relations

$$\mathcal{P}_i(t) = p_i \mathcal{E}(t).$$




$t_i = -20,\ p_1 = p_2 = \frac{2}{3},\ p_3 = -\frac{1}{3}$

# Theoretical explanation (I)

**This near-singularity universality can be understood theoretically.**

Argument (for simplicity, $M = 0$):

- Assume $\langle T_{\mu\nu}\rangle \sim (-t)^{\gamma-4}$, with undertermined $\gamma < 4$.
- Solve bulk Einstein equations in a near-boundary, $1/r$-expansion,

$$A = r^2 - \frac{\kappa|t|^{\gamma-4}}{r^2} + \frac{\kappa(\gamma-4)|t|^{\gamma-5}}{2r^3} - \frac{9\kappa(\gamma-4)^2|t|^{\gamma-6}}{40r^4} + \ldots,$$

$$|t|^{-\frac{1}{3}}\Sigma = r + \frac{1}{3t} - \frac{1}{9t^2 r} + \frac{5}{81t^3r^2} - \frac{10}{243t^4r^3} + \frac{22}{729t^5r^4} + \ldots + \kappa\left(-\frac{(3\gamma-8)|t|^{\gamma-5}}{60r^4} + \frac{(176-108\gamma+15\gamma^2)|t|^{\gamma-6}}{360r^5} - \frac{(-18976+15564\gamma-3930\gamma^2+315\gamma^3)|t|^{\gamma-7}}{15120r^6} + \ldots\right),$$

$$B_i = b_i\left(\log|t| + \frac{1}{tr} - \frac{1}{2t^2r^2} + \frac{1}{3t^3r^3} - \frac{1}{4t^4r^4} + \ldots\right) + \lambda_i\left(-\frac{|t|^{\gamma-4}}{r^4} + \frac{(\gamma-4)|t|^{\gamma-5}}{r^5} - \frac{(\gamma-4)(7\gamma-34)|t|^{\gamma-6}}{12r^6} + \ldots\right) + b_i\kappa\left(-\frac{|t|^{\gamma-5}}{5r^5} - \frac{(\gamma-5)|t|^{\gamma-6}}{5r^6} + \ldots\right),$$

Splitting into a $\gamma$-independent and a $\gamma$-dependent piece.

# Interlude: the vacuum solution

- The $\gamma$-independent piece is a solution of five-dimensional Einstein gravity with a negative cosmological constant with $\langle T_{\mu\nu} \rangle = 0$: *vacuum solution*.

$$A_0(t,r) = r^2, \quad \Sigma_0(t,r) = (-t)^{\frac{1}{3}} r \left(1 + \frac{1}{tr}\right)^{\frac{1}{3}}, \quad B_{i,0}(t,r) = b_i \log(-t) + b_i \log\left(1 + \frac{1}{tr}\right).$$

- In FG coordinates, the vacuum solution is simply

$$ds^2 = \frac{d\mu^2 + ds_K^2}{\mu^2}.$$

There is always such a five-dimensional uplift with zero stress-energy tensor of a four-dimensional Ricci-flat solution

- The vacuum solution has a singularity at $r_s = -1/t$, an AH at $r_{AH} = -4/(3t)$, and an EH at $r_{EH} = -2/t$.

$$r_s \leq r_{AH} \leq r_{EH}$$

# Theoretical explanation (II)

- The near-boundary expansion has the functional form

$$\frac{A}{A_0} = 1+(-t)^{\gamma} A_c\left(-\frac{1}{tr}\right), \quad \frac{\Sigma}{\Sigma_0} = 1+(-t)^{\gamma}\Sigma_c\left(-\frac{1}{tr}\right), \quad B_i - B_{i,0} = (-t)^{\gamma} B_{i,c}\left(-\frac{1}{tr}\right).$$

- To find $\gamma$, we introduce the scaling variable $\zeta = -1/(tr)$, and solve the bulk Einstein equations in the $t \to 0^-$ limit at finite $\zeta$.

➤ The bulk Einstein equations reduce to a third-order master ODE in $\zeta$ for $\Sigma_c$, and an additional second-order ODE in $\zeta$ for $b_1 B_{1,c} - b_2 B_{2,c}$ source by $\Sigma_c$.

➥ Demanding that $\Sigma_c$ has the right asymptotic behavior and is regular at the EH of the vacuum solution uniquely fixes $\gamma = 2$ and leads to

$$\langle T_{\mu\nu}\rangle \sim (-t)^{\gamma-4} = (-t)^{-2}.$$

➥ Similarly, noting that $\gamma = 2$ and demanding that $b_1 B_{1,c} - b_2 B_{2,c}$ has the right asymptotic behavior and is regular at the EH of the vacuum solution allows to express $\lambda_i$ in terms of $\kappa$ and leads to

$$\mathcal{P}_i = p_i \mathcal{E}.$$

## Comments

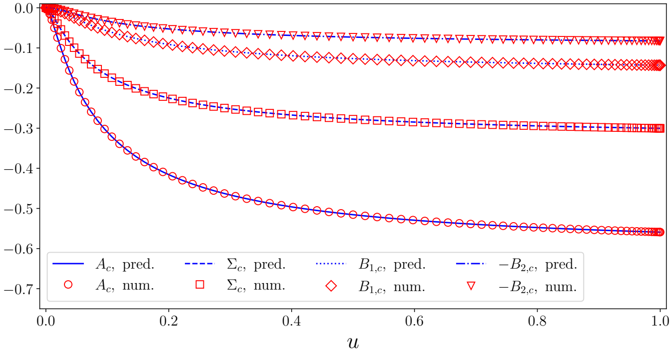
- The argument can be simply generalized to $\phi \neq 0$, with identical main results.
- Note that $(-t)^\gamma \times \{A_c(\zeta), \Sigma_c(\zeta), B_{1,c}(\zeta), B_{2,c}(\zeta)\}$ act as linearized perturbations of the vacuum solution in the $t \to 0^-$ limit at finite $\zeta$: **we are explaining a divergent boundary one-point function in terms of a linearized bulk gravity computation**.
- This computation is morally analogous to the one of Janik & Peschanski in 2005 to find the late-time hydrodynamic behavior of holographic Bjorken flow, however here we are zooming in into short-time physics.

# Final crosscheck

$$A_c = -\frac{2}{3}\zeta((M^2 - 18\kappa)\zeta - 9\kappa(2-\zeta)\log(1-\zeta)),$$

$$\Sigma_c = \frac{\zeta(9\kappa(12 - 12\zeta + \zeta^2) - M^2\zeta(9 - 7\zeta)) + 54\kappa(2 - 3\zeta + \zeta^2)\log(1-\zeta)}{27(1-\zeta)},$$

$$B_{i,c} = -b_i\frac{\zeta(9\kappa(6 - 15\zeta + 5\zeta^2) + 8M^2\zeta^2) + 54\kappa(1-\zeta)(1-2\zeta)\log(1-\zeta)}{36(1-\zeta)}.$$




$u = 1 + \sqrt{3}$, comparison time $-0.033$

## Message III: perturbative backreaction suggests the BKL behavior might be avoided

# Perturbative backreaction (I)

- Since $\langle T_{\mu\nu} \rangle$ diverges universally as the cosmological singularity is approached, **its perturbative backreaction on the four-dimensional Kasner metric should also exhibit universality.**

➤ Relevant parameter: effective gravitational coupling

$$\mathcal{G} = N^2 G^{(4)}$$

➤ We work in the $G^{(4)} \to 0$, $N \to \infty$ limit with $\mathcal{G}$ fixed and perturbatively small.

$$G^{(4)}_{\mu\nu} = 8\pi\mathcal{G}\langle T_{\mu\nu} \rangle$$

$$g^{(4)}_{\mu\nu} = g^{(4)}_{K,\mu\nu} + \mathcal{G}\delta g^{(4)}_{\mu\nu} + O(\mathcal{G}^2), \quad \langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_K + \mathcal{G}\delta\langle T_{\mu\nu} \rangle + O(\mathcal{G}^2)$$

➤ At leading nontrivial order, we have to solve the boundary Einstein equations linearized around the Kasner metric with the holographic stress-energy tensor as a source.

# Perturbative backreaction (II)

- Crucial point: since $\langle T_{\mu\nu}\rangle \sim t^{-2}$, the background $\log g_{ii}(t) = 2p_i \log(-t)$ gets a leading order nontrivial perturbative correction with the same $\log(-t)$ time dependence.

➥ **This correction can be reabsorbed into a perturbative redefinition of the Kasner exponents $p_i$.**

$$p_1 + p_2 + p_3 = 1 + 4/\pi\Lambda\mathcal{G}, \quad p_1^2 + p_2^2 + p_3^2 = 1.$$

$$\Lambda \equiv \lim_{t\to 0^-} t^2\mathcal{E}(t)$$

(Compare with the classical massless scalar field, $p_1 + p_2 + p_3 = 1$ & $p_1^2 + p_2^2 + p_3^2 = 1 - 16\pi G\Lambda$)

➥ If $\Lambda > 0$, **the correction opens a novel window where the three Kasner exponents can be positive.**

➥ Having all $p_i \geq 0$ will destroy the instability driving the BKL transitions and end the BKL dynamics after a finite number of epochs.

More research is needed to explore whether this is actually the case!

# Future directions

- We can immediately generalize the present analysis to Bianchi type II (Kasner-to-Kasner transition), Bianchi type IX (mixmaster universe, full BKL dynamics).
- Build a self-consistent semi-holographic framework to treat finite $\mathcal{G}$.
- Full vacuum solution feautures naked singularities, but here only a fraction of it becomes relevant dynamically. Suggests a role for similar five-dimensional uplifts of four-dimensional Ricci flat geometries more generally?

Thanks!