YANG-MILLS-LIOUVILLE THEORY

STRINGTHEORY.PL
UJ, 7.06.2024

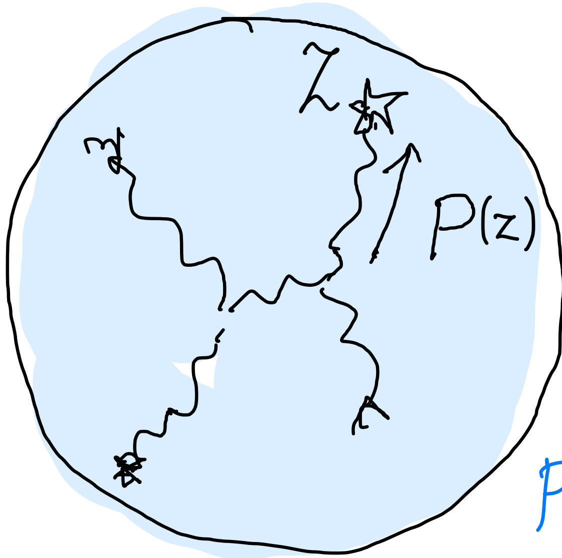
TOMASZ TAYLOR
UW/NEU

work with Stephan Stieberger & Bin Zhu
2022-24

YM observables: scattering amplitudes

↕ Celestial map

L observables: primary field correlators
on (celestial) sphere



$p^2 = m^2 = 0$

Celestial Map

Pasterski, Shao, Strominger

momentum $p^\mu = \omega q^\mu$

$$q^\mu = \frac{1}{2}\left(1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2\right)$$

ω is energy

z specifies direction of p^μ

= point on Celestial Sphere

Lorentz symmetry $SL(2,\mathbb{C})$

$$z \to \frac{az+b}{cz+d}$$

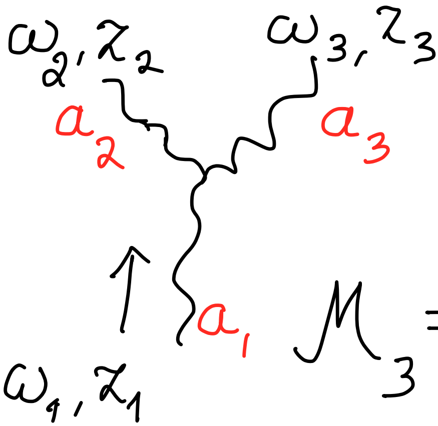
$$\omega \to |cz+d|\,\omega$$

Celestial sphere: natural arena for CFT:

Celestial Conformal Field Theory

Example:

3-gluon MHV



$$\mathcal{M}_3 = g\, f_{a_1 a_2 a_3}\, \frac{\omega_2\,\omega_3}{\omega_1}\, \frac{(z_2 - z_3)^3}{(z_1 - z_2)(z_3 - z_1)}$$

"color factor"

CFT primary field operators $\underset{\substack{\Delta \\ \uparrow \text{dimension}}}{\mathcal{O}}(z, \bar{z})$

No room for ω : trade it for Δ

Celestial amplitudes

$$\tilde{A} = \underbrace{\int d\omega_1 \, \omega_1^{\Delta_1 - 1} \cdots d\omega_n \, \omega_n^{\Delta_n - 1}}_{\text{Mellin transform (changes } e^{ipx} \text{ to boost basis)}} \delta^{(4)}\left(\sum p_{out} - \sum p_{in}\right) \mathcal{M}(\omega_i, z_i, \bar{z}_i)$$

$$= \left\langle \mathcal{O}_{\Delta_1}(z_1) \cdots \mathcal{O}_{\Delta_N}(z_N) \right\rangle$$

gauge charge

$$\text{on-shell particle } (p^2=0) \longleftrightarrow \mathcal{O}^{a}_{\Delta, h}(z,\bar{z})$$

helicity

$$\text{YM} \longleftrightarrow \text{Liouville}$$

Liouville Theory

2D QG theory of conformal factor

Liouville field

$$\text{CFT}: \quad \mathcal{L} = \frac{1}{\pi}\left|\frac{\partial\phi}{\partial z}\right|^2 + \mu e^{2b\phi}$$

coupling constant

2D $z, \bar{z}$ coordinates

cosmological constant

Fundamental work of Dorn, Otto & Zamolodchikov^2 DOZZ (1994-6)

$$b + \frac{1}{b} = Q \qquad\qquad C = 1 + 6Q^2$$

background charge

central charge

$b \to 0$ $[C \to \infty]$ limit is nontrivial ($b \to \frac{1}{b}$ duality)

Primary field operators $V_\alpha(z,\bar{z}) = e^{2\alpha\phi(z,\bar{z})}$

dimension $\Delta = 2\alpha(Q-\alpha)$

$\langle V_{\alpha_1} V_{\alpha_2} V_{\alpha_3} \rangle$ = DOZZ formula, exact!

special case $\alpha = \sigma b$: light operators

Weak Liouville coupling $b \to 0$:

correlators of light operators can be computed by using semiclassical approximation

CCFT for (perturbative) Yang-Mills

$$\text{WZW} \times \text{Liouville} \begin{pmatrix} b \to 0 \\ c \to \infty \end{pmatrix}$$

helicity

group index

$$\underset{\Delta = 1 + i\lambda}{\mathcal{O}^{+a}(z)} = \underset{\Delta = 1}{J^a(z)} \; \underset{\Delta = i\lambda}{e^{i\lambda b \phi(z,\bar{z})}}$$

$$\underset{\Delta = i\lambda}{\mathcal{O}^{-a}} = \underset{\Delta = -1}{\hat{J}^a(z)} \; \underset{\Delta = 1 + i\lambda}{e^{(1+i\lambda) b \phi(z,\bar{z})}}$$

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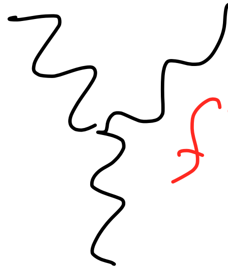
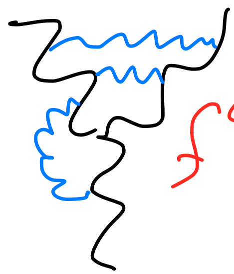
As $b \to 0$

$$\langle \mathcal{O}^{a_1} \mathcal{O}^{a_2} \mathcal{O}^{a_3} \rangle =$$  $f^{a_1 a_2 a_3}$

YM Tree level $(g \to 0)$

loop corrections

 $f^{a_1 a_2 a_3}$

Known EXACTLY

Zamolodchikov^2

to ALL orders in b!

Known to many orders

powers g^n

2. Liouville three-point function

(Zamolodchikov)

The three-point function of exponential fields $\exp(2a\phi)$

$$\left\langle e^{2a_1\phi}(x_1)e^{2a_2\phi}(x_2)e^{2a_3\phi}(x_3)\right\rangle_{\mathrm{L}} = \frac{C_{\mathrm{L}}(a_1,a_2,a_3)}{(x_{12}\bar{x}_{12})^{\Delta_1+\Delta_2-\Delta_3}(x_{23}\bar{x}_{23})^{\Delta_2+\Delta_3-\Delta_1}(x_{31}\bar{x}_{31})^{\Delta_3+\Delta_1-\Delta_2}} \tag{2.1}$$

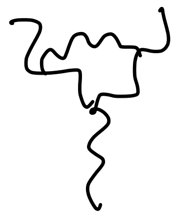
in Liouville field theory has been discovered by Dorn and Otto [7] in 1992. The coordinate dependence of (2.1) involves the dimensions $\Delta_i = \Delta_{a_i}$ of the exponential fields given by eq.(1.12). The dependence is standard and therefore we'll omit this multiplier and call the factor $C_{\mathrm{L}}(a_1,a_2,a_3)$ the three-point function. In the notations of ref. [9] it reads explicitly

$$C_{\mathrm{L}}(a_1,a_2,a_3) = \left(\pi\mu\gamma(b^2)b^{2-2b^2}\right)^{(Q-a_1-a_2-a_3)/b} \times \frac{\Upsilon(b)\Upsilon(2a_1)\Upsilon(2a_2)\Upsilon(2a_3)}{\Upsilon(a_1+a_2+a_3-Q)\Upsilon(a_1+a_2-a_3)\Upsilon(a_2+a_3-a_1)\Upsilon(a_3+a_1-a_2)} \tag{2.2}$$

Here $\Upsilon(x) = \Upsilon_b(x)$ is a special function related to the Barnes double gamma function [8] (see [9] for the precise definitions and properties). It should be noted at this point that (2.2) is unnormalized correlation function. To use it in the developments like (1.17) one has to divide it by the Liouville

Assume it is a Mellin transform of "EXACT" YM amplitude

Compare $O(b^2)$ correction with



$\sim \log Q^2$

$$b^2 \approx \frac{g^2(M)}{8\pi^2}\beta_0 \quad \leftarrow \frac{11 N_{\text{colors}}}{3}$$

"EXACT" ?

gluons confined

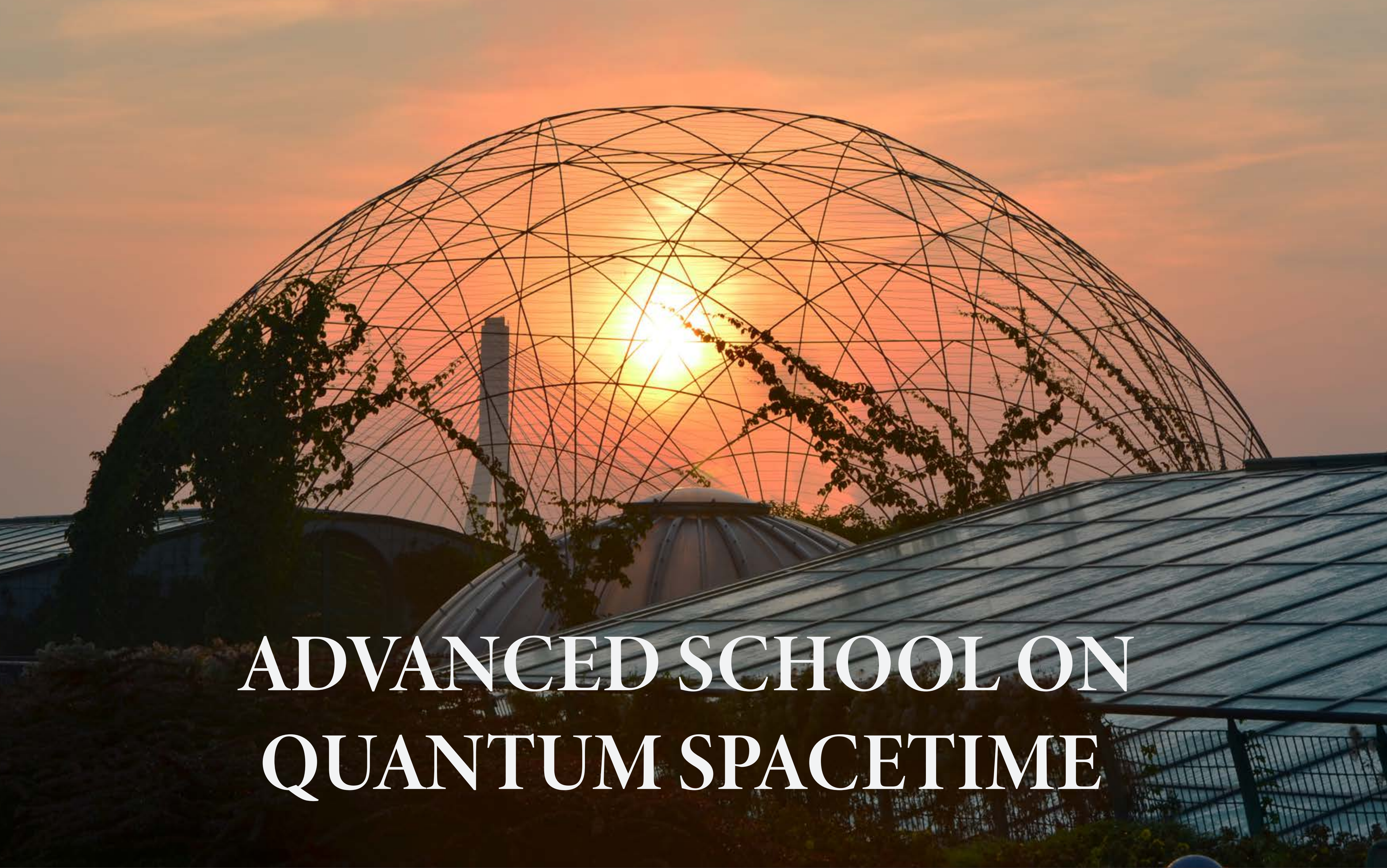
mass gap - glueballs?

The only way to answer this question is to take INVERSE Mellin of EXACT WZW-Liouville correlators

... perhaps numerically?

...IN PROGRESS 

DZIĘKUJĘ!



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