Holographic evolution of timelike entanglement entropy

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based on work in progress with Fabio Ori and Alex Serantes

Introduction

Top down sharp geometric probes of the bulk

hep-th/9906226 by Balasubramanian and Ross, hep-th/0603001 by Ryu and Takayanagi, ...

Holography encodes gravity in the language of boundary CFTs

What we know about the bulk ultimately stems from $Z_{\text{bulk}}[J] = Z_{\text{CFT}}[J]$

This gives us direct access to correlation functions and thermodynamic qties

While all these qties are geometric, they are only sometimes sharp in the bulk



What they are good for in the bulk?

They are also bottom up sharp geometric object, like holographic complexity, but these are in the vast majority of cases at best only qualitatively understood



 $\mathcal{C}_{V} \sim \text{volume of } \max_{\min} (\text{Lorentzian}) \text{ volume time slice}$ $\mathcal{C}_{A} \sim \text{bulk action in the Wheeler - de Witt patch}$ $\mathcal{C}_{V 2.0} \sim \text{bulk volume of the Wheeler - de Witt patch}$ $\mathcal{C}_{anything} \sim \underset{\text{using a whole class of functionals}}{\operatorname{condense}}$

1402.5674 by Susskind, 1509.07876 by Brown et al., 1610.02038 by Couch et al., ..., 2111.02429 by Belin et al.

We like such sharp quantities because due to their localized nature they allow to directly probe black hole regions of interests (horizon, interior, singularity)

And if they are top down and we could calculate them independently on the boundary, the match with the bulk representation indicates bulk geometry works

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And if they are top down and we could calculate them independently on the boundary, the match with the bulk representation indicates bulk geometry works

What they are good for on the boundary?

Top down:

[statics] Irreversibility of renormalization group flows (c-, F-, a-theorems)

[dynamics] Thermalization at strong coupling as seen by nonlocal observables

[dynamics] Entanglement entropy production in quenches at strong coupling

Most often botton up:

[dynamics] growth of circuit/state/Krylov (?) complexity at strong coupling



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2305.11280 with Aguilar and Van der Schueren





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Timelike entanglement entropy

2210.09457, 2302.11695 by Doi, Harper, Mollabashi, Takayanagi, Taki



Why this talk?

Tempting bulk picture based on a comparison with CFT₁₊₁ analytic continuations: 2210.09457, 2302.11695 by Doi, Harper, Mollabashi, Takayanagi, Taki





$$\frac{c}{3}\log\frac{\sqrt{(t_1-t_2)^2}}{\epsilon} + \frac{c}{3}\left(i\frac{\pi}{2}\right)$$

Our aim: providing the prescription and studying its properties and implications work in progress with Fabio Ori and Alex Serantes

Our proposal

Our proposal work in progress with Fabio Ori and Alex Serantes



(holographic) timelike entanglement entropy $S = \frac{\text{proper area}}{4G_N} \text{ of a complex}$ boundary anchored extremal surface of codimension 2 with the smallest **Re(proper area)**

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Extremal surface means we extremize the surface function no matter what

Complex means the surface lives in the bulk metric $g_{ab}(x^c)$ with x^c complexified

Boundary anchored means that the surface satisfied real boundary conditions, e.g.



Applications

Vacuum in CFT_{I+I} work in progress with Fabio Ori and Alex Serantes

Complex geodesic reproduces analytic continuation of the CFT₁₊₁ result



A necessary feature work in progress with Fabio Ori and Alex Serantes

We can take different paths in the complex affine parameter plane



Here only a few (2?) can be interpreted in terms of paths in real geometries

Indeed, analytic continuation of strip entanglement entropy in hCFT₁₊₂ does not seem to have <u>any</u> interpretation in terms of paths in real bulk

However, for our proposal it does not matter

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Quenches work in progress with Fabio Ori and Alex Serantes

This way of viewing things allow us to study also holographic quenches in CFT_d



Timelike entanglement entropy in quenches

work in progress with Fabio Ori and Alex Serantes

CFT_{I+I} and $\gamma \rightarrow \infty$: two ways of doing things:

- matching exact solutions at v = 01212.6066 by Balasubramanial et al.

- fully fledged numerics



Otherwise, it is numerics in complexified Vaidya, pilot results in 4 bulk dimensions

Summary



Sharp and well defined observables are rare and precious (bulk tomography)

Brilliant idea in 2022/2023: analytic continuation of holographic entanglement entropy to timelike subregions might as well be such a quantity 2210.09457, 2302.11695 by Doi, Harper, Mollabashi, Takayanagi, Taki

Us: providing a bulk prescription and studying geometric interpretation work in progress with Fabio Ori and Alex Serantes

Take home: holographic timelike entanglement entropy is necessarily given by complex extremal codimension-2 hypersurfaces

work in progress with Fabio Ori and Alex Serantes



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Outlook



Replica trick derivation?

Holographic timelike entanglement entropy as probe of black hole singularity?



Physical interpretation? Perhaps temporal entanglement in TNS

Tip of the iceberg of novel bottom up holographic geometric probes?



