# *Subregion Complexity of Islands in black hole models*


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*08 June , 2024*

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*Based on arXiv:2010.04134, JHEP 02 (2021) 152*
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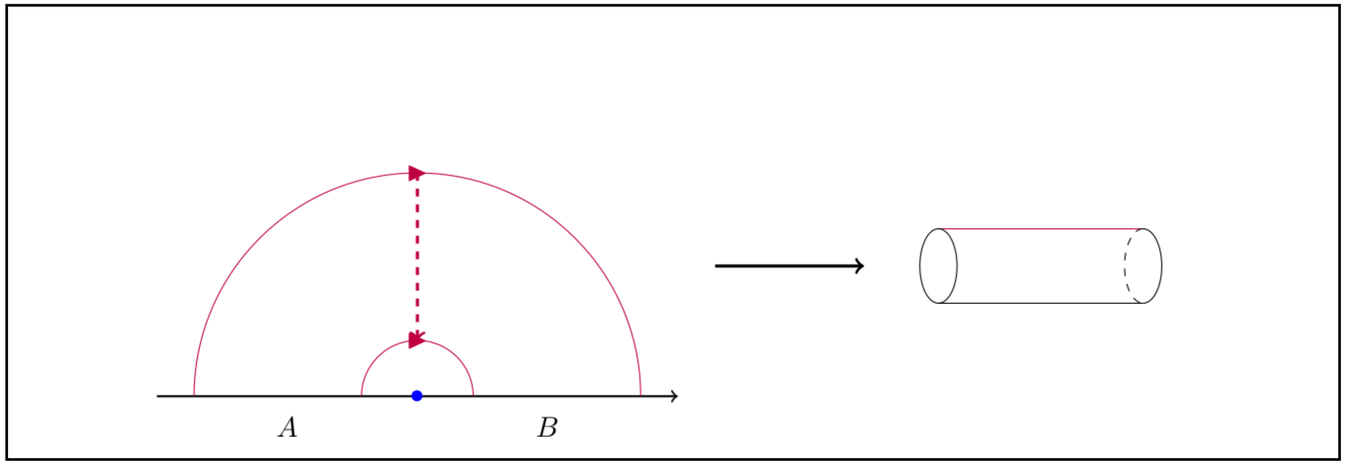
funded by
NARODOWE CENTRUM NAUKI
grant 2021/42/E/ST2/00234

# MOTIVATION

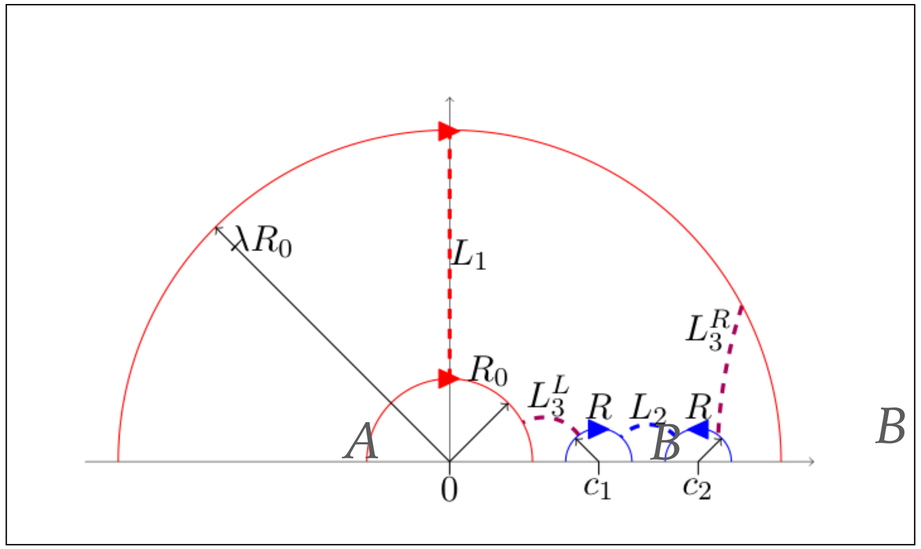
- BH information paradox→ a long-standing problem in theoretical physics.
- AdS/CFT and quantum information →Ryu-Takayanagi conjecture →Engelhardt-Wall QES→resolution?
- Is (BH+Radiation) a unitary system? (Expectation→ Page curve)
- Rescuer → QES & Islands (bulk regions included in the entanglement wedge of the radiation subsystem at Page time). (Almheiri et al 2019, Pennington 2019)
- What exactly happens at Page time physically? →What do (/can) we learn from simple models?

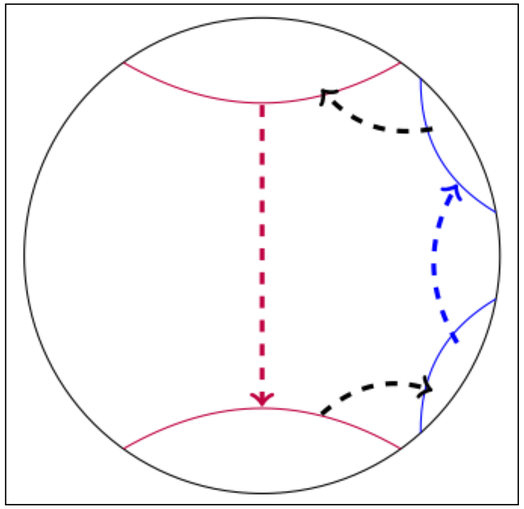
# MULTIBOUNDARY WORMHOLES

- Empty $AdS_3$ → solution of Einstein equation → constant negative curvature → maximally symmetric.
- Variants in 3d →remove symmetries....how? →Quotienting timeslice by discrete groups.
- Two sided BTZ →identifying two concentric semicircles on the upper half plane with same orientation. Remember → semicircles are geodesics...each can have two orientations.



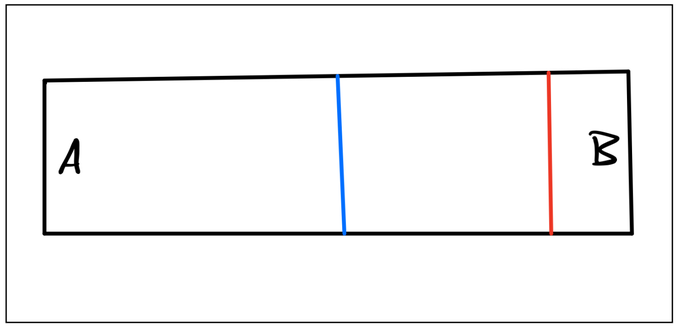
- Increase number of exits?→ Remove more symmetries → identify more semicircles,but not concentric. For each boundary, identify two semicircles with opposite orientations on one side of the smaller concentric semicircle.
- $L_2$ (connected) and $L_3$ (disconnected) new throat horizons.
- Lengths of $L_{1,2,3}$ can be tuned independently.
- Natural playground to study multipartite entanglement (Balasubramanian et al, 2013)

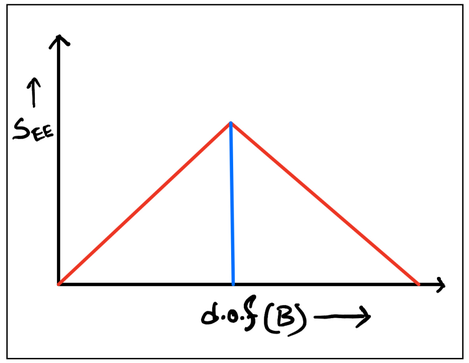


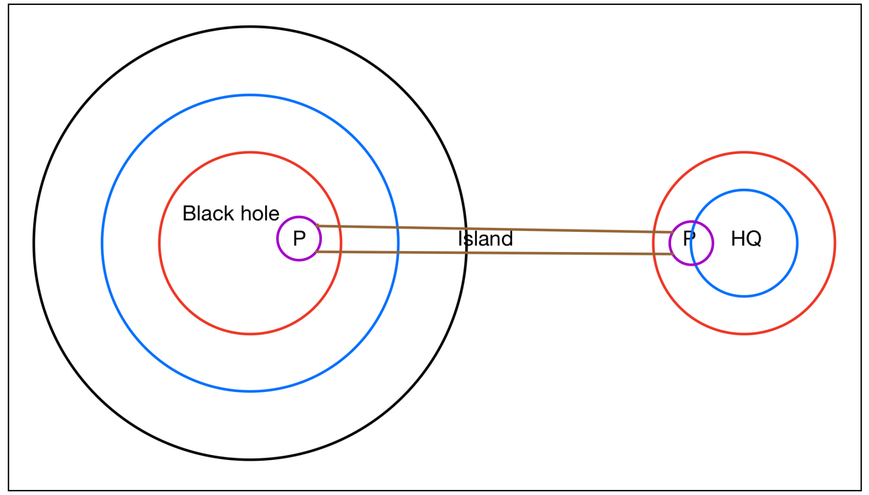



# MODEL OF EVAPORATION

- Unitary system → Page curve
- Page time → system and compliment has same # of d.o.f.
- Evaporating BH (system) → Radiation quanta (compliment)
- Page time →purification of certain Hawking modes→ Island inclusion.

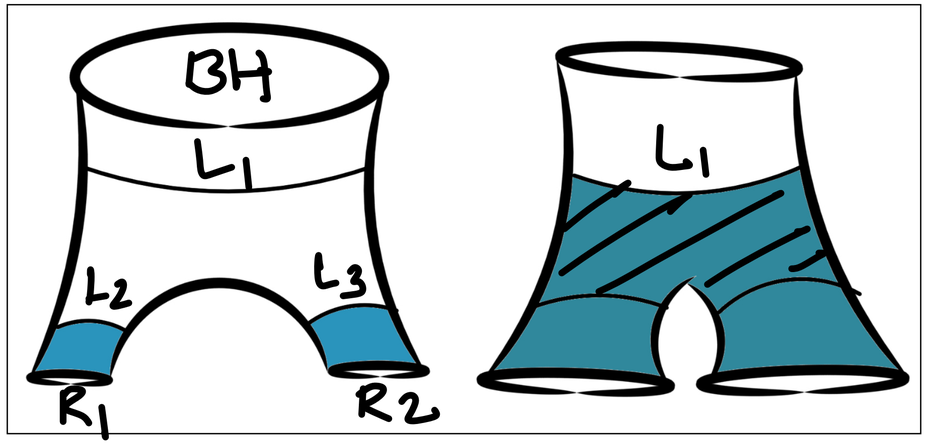


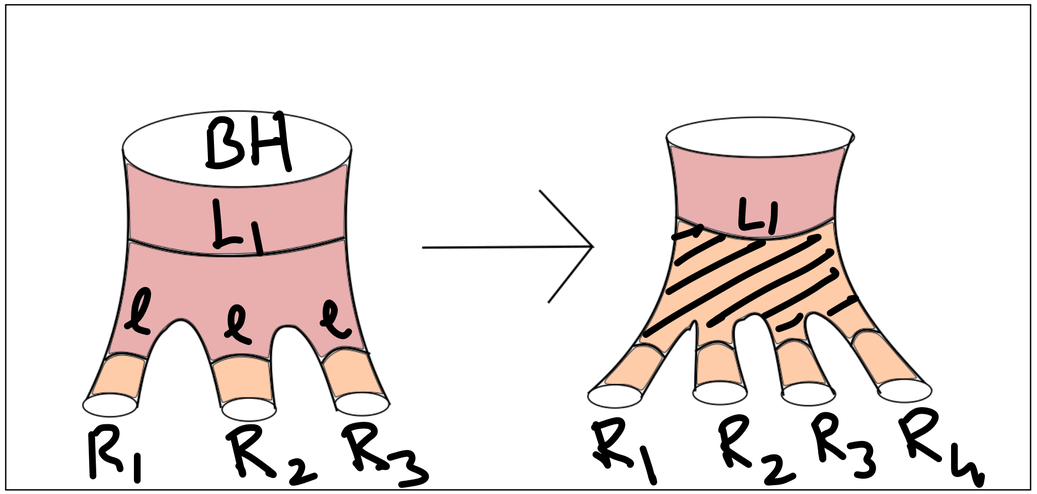
# MODEL OF EVAPORATION - I

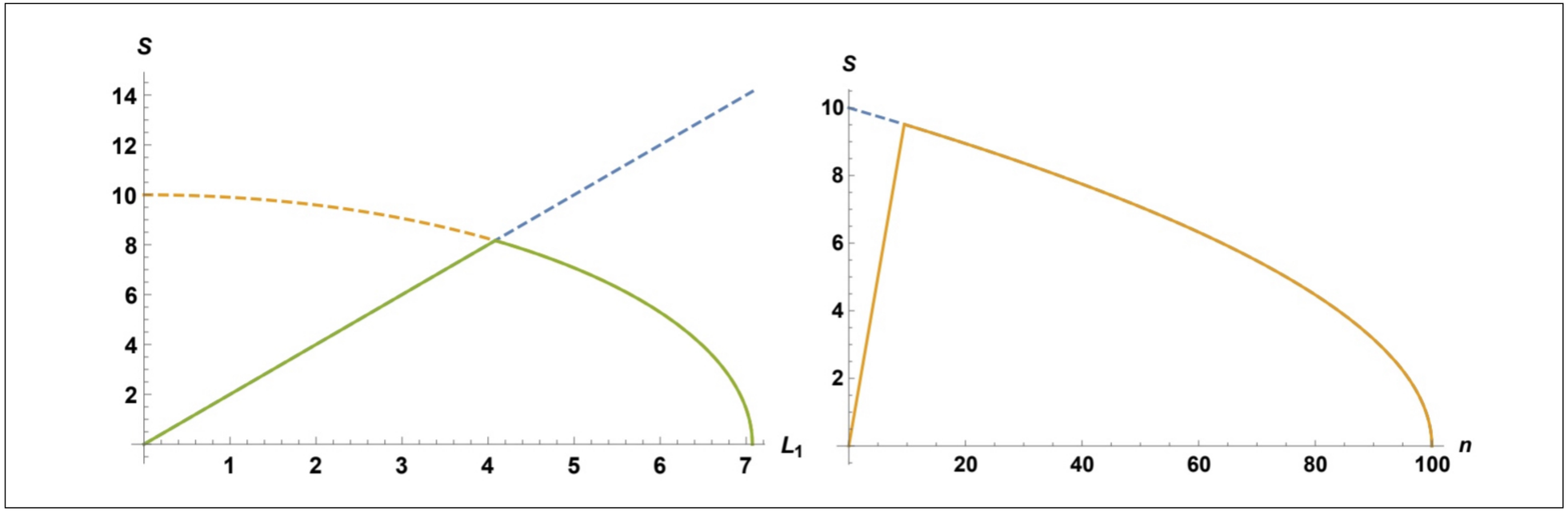
- Three boundary wormhole → bigger exit is black hole, smaller exits are radiation storages. (Zhou ' 2020)
- Black hole evaporates ($L_1 \downarrow$ ) → Storage size increases ($L_{2,3} \uparrow$)
- Throat horizons are candidate HRT surfaces ($L_1$ and Union of $L_{2,3}$).
- Energy entropy relation in $AdS_3$ results in relating time dependencies of the two candidate HRT surfaces between radiation and BH exits. $L_1(t)^2 = L_1(0)^2 - \left(L_2(t)^2 + L_3(t)^2\right)$
- Simplifying relation → $L_1(t) = \sqrt{L_1(0)^2 - 2L_2(t)^2}$



# MODEL OF EVAPORATION – II

- (n+1) boundary wormhole → shrinking bigger exit is black hole. Increasing # of very small exits are multipartitely modelled radiation quanta. n instead of two.
- Topology changes at each timestep →each snapshot is time reflection symmetric → not a solution of EE dynamically, simply a model (Harlow, Engelhardt, Akers '19).
- Number of exits is the representative of time.
- All the smaller exits are assumed to be of the same size $L_{2,\ldots,(n+1)} = \ell$.
- Information conservation equation $\rightarrow L_1(n) = \sqrt{L_1(0)^2 - n\ell^2}$.

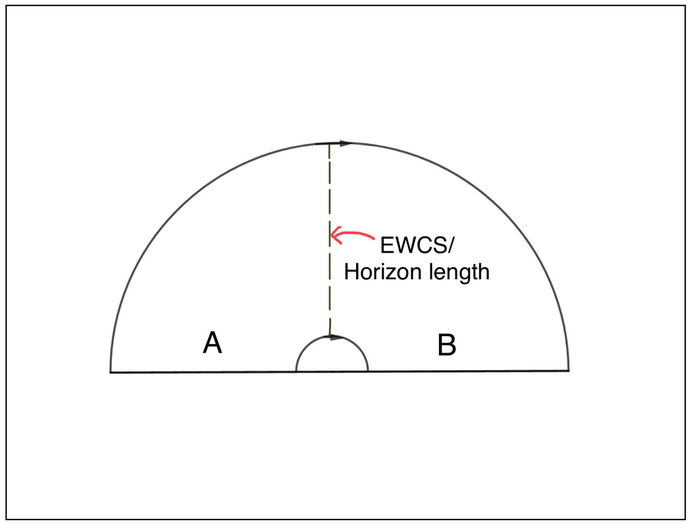


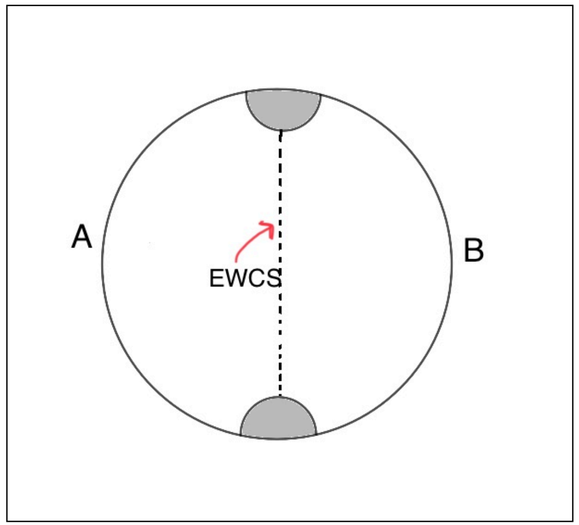


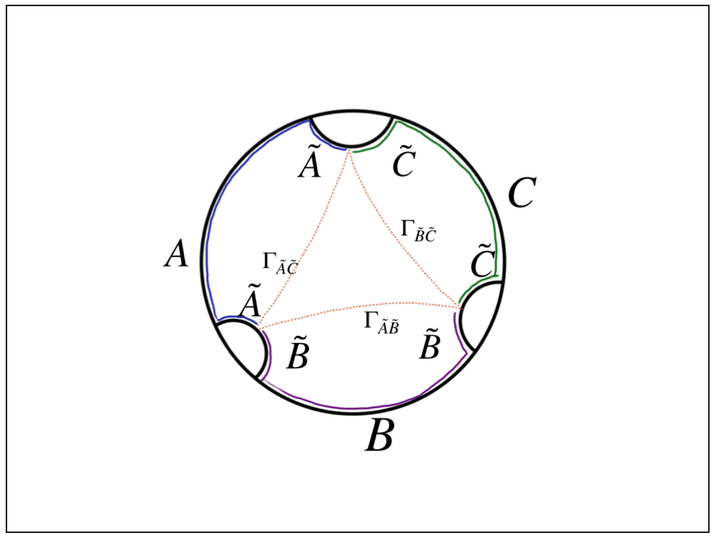

- In the Multiboundary wormhole models, the causal shadows play the role of the islands.
- The islands are arguably connected to quantum error correction (QEC).
- Related the boundary of the causal shadow to the holographic proposal of multipartite EoP. (AB, 2003.11870, PRD)
- Purification: Given a mixed state, add ancillary d.o.f to make the state pure→ infinitely many ways → choose a particular one that minimises the QI measure of interest.

# ENTANGLEMENT OF PURIFICATION

- Entanglement of purification is an information theoretic quantity measuring entanglement between parts of a mixed state.
- AB being a mixed state and for all the choices of ancillary system $A'B'$ so that the total system is pure, EoP $\rightarrow$ minimised Von Neumann Entropy between $AA'$and $BB'$.
- Holographic proposals of bipartite EoP (Takayanagi, Umemoto '2018)



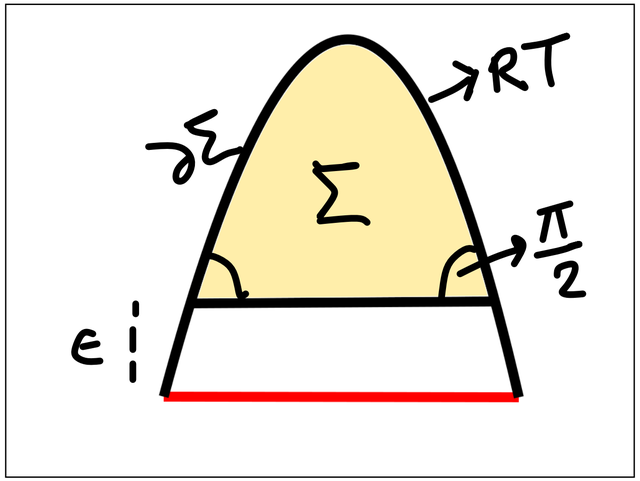



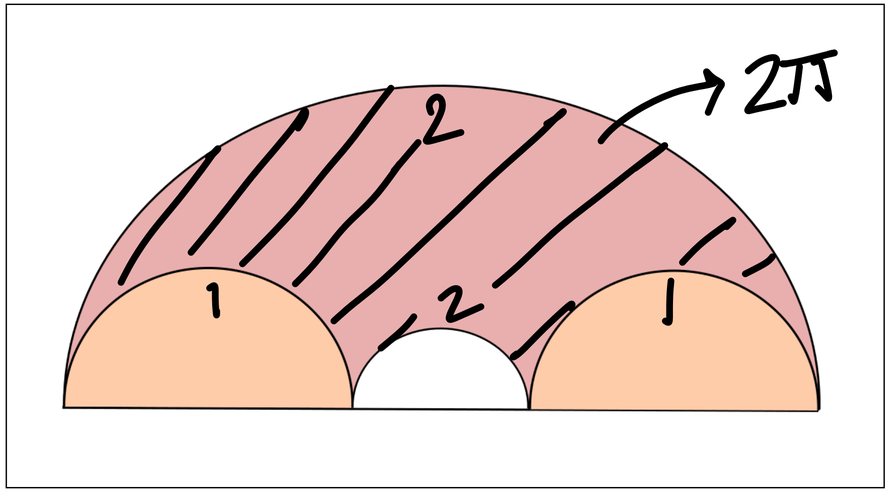



# VOLUMES (COMPLEXITY) IN ADS3

- Holographic proposal → Maximal volume slice = Complexity (measure of the number of elementary quantum gates needed to prepare a quantum state) (Susskind)
- Volume dual to RT surface → Mixed state(sub-region) complexity (Alishahiha '15)
- Topological complexity in $AdS_3$ (Erdmenger et al '17)
- Geodesic curvature and Euler characteristics of the bulk region contribute to the volume. $C(A) = -\frac{1}{2}\int_{\Sigma} R d\sigma = \int_{\partial\Sigma} k_g ds + \sum_{i=1}^{r} \alpha_i - 2\pi\chi(\Sigma).$
- Note → Bulk geodesics make an angle $\frac{\pi}{2}$ reaching the boundary.
- What happens when the HRT choice changes?

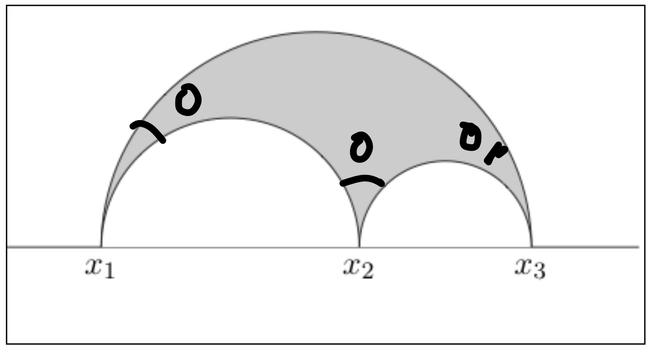
- Two scenarios where the HRT change → i) empty $AdS_3$ disjoint subregions, ii) BTZ mass parameter change.

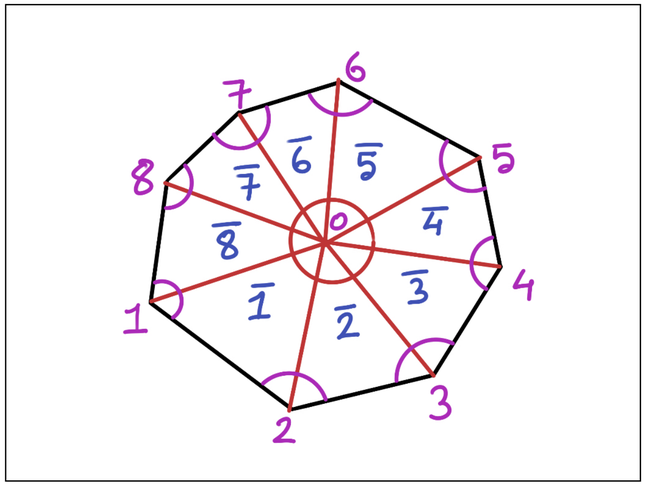


# COMPLEXITY OF ISLANDS

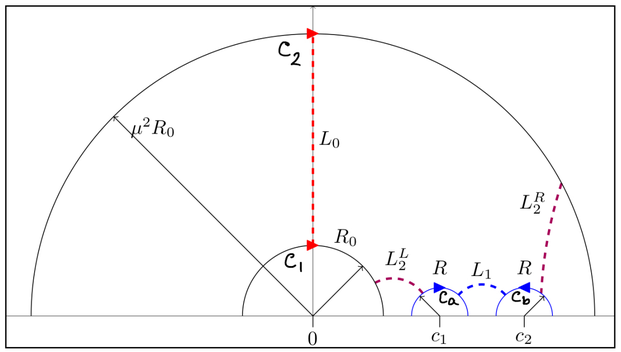
- Compute volumes dual to HRTs (throat horizons) corresponding to radiation exits in the multiboundary wormholes (fundamental domain).
- Causal shadow regions get added at the point of HRT change (Page time).
- Two models need quantitatively different treatment →notion of time is different.
- Volume of causal shadow → area of hyperbolic polygon (use Gauss-Bonnet theorem).

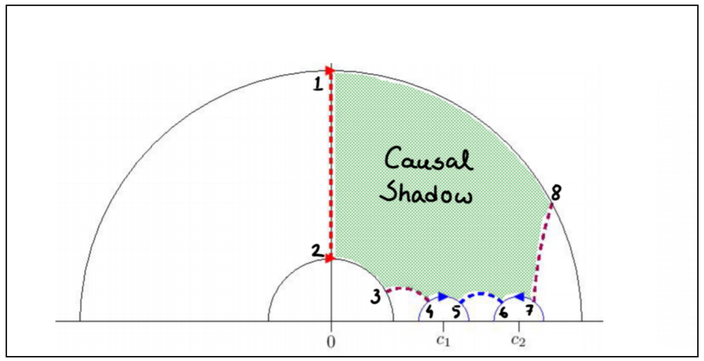


# CAUSAL SHADOW(ISLAND)

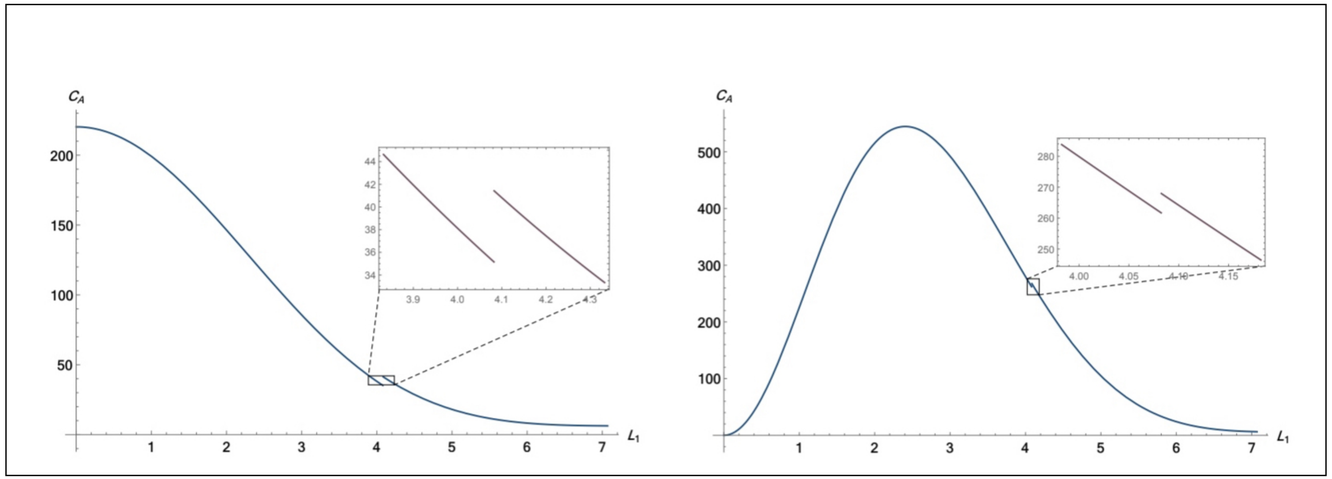
- In case of the three bdy model, the causal shadow is a hyperbolic octagon.
- All the vertices have bulk geodesics meeting.
- Area of the octagon is $2\pi$.
- In case of the n bdy model, the causal shadow is a hyperbolic $4(n_{page} - 1)$ gon.
- Area of the $4(n_{page} - 1)$ gon is $2(n_{page} - 2)\pi$.

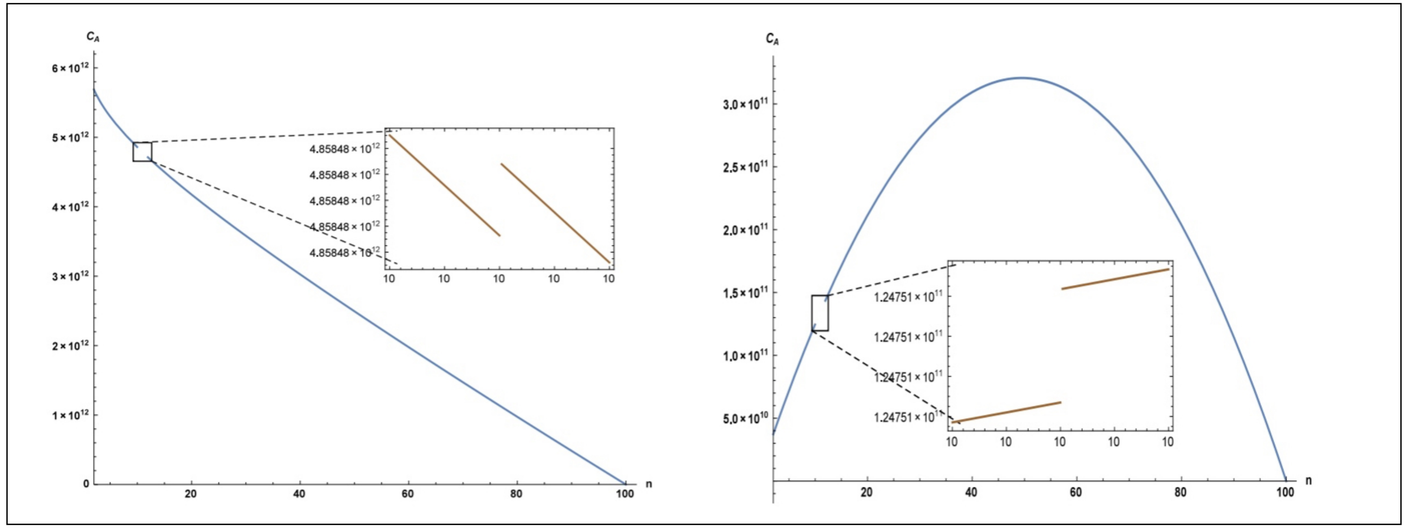


# “LONDON BRIDGE FALLING DOWN” AND “WHAT GOES UP, MUST COME DOWN”

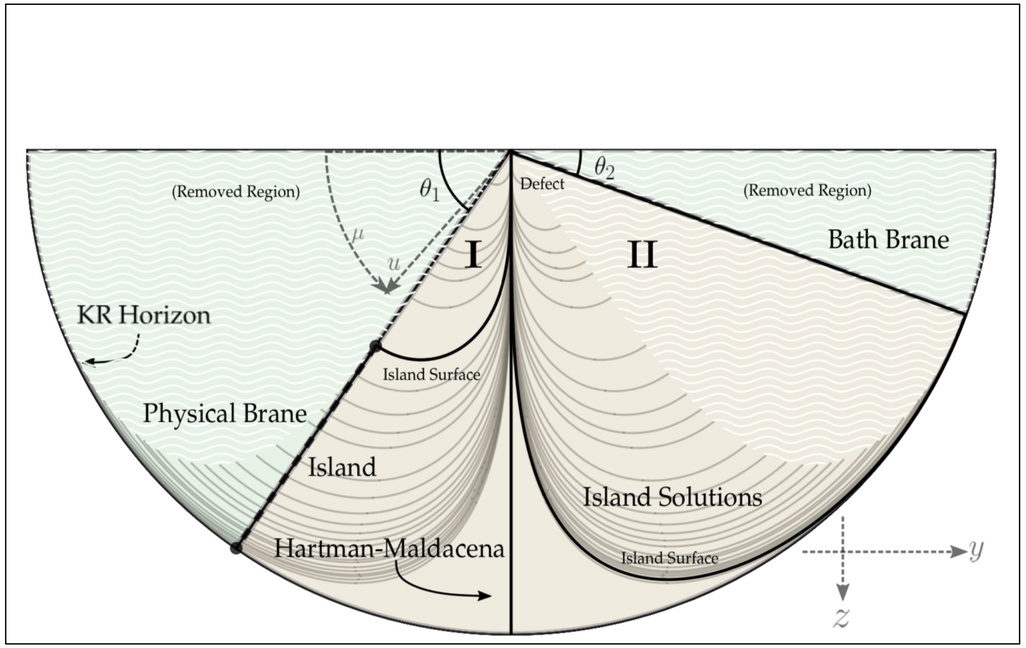
- Three boundary & n-boundary model volume plots

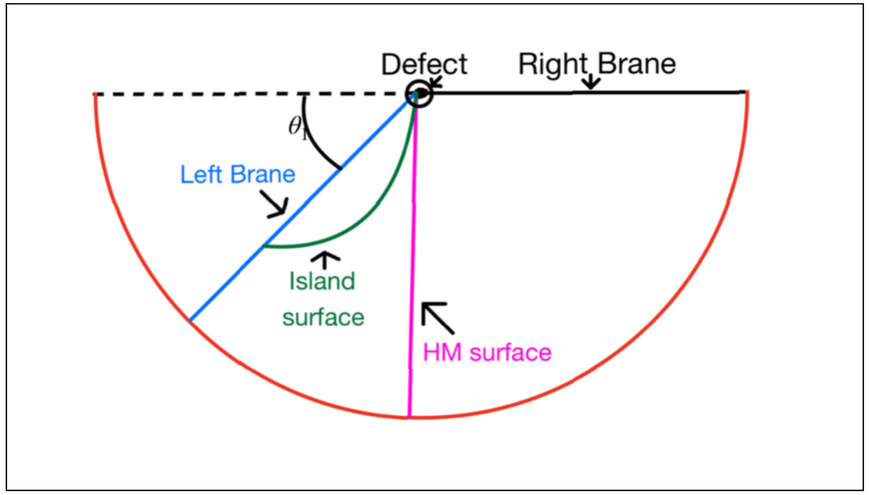


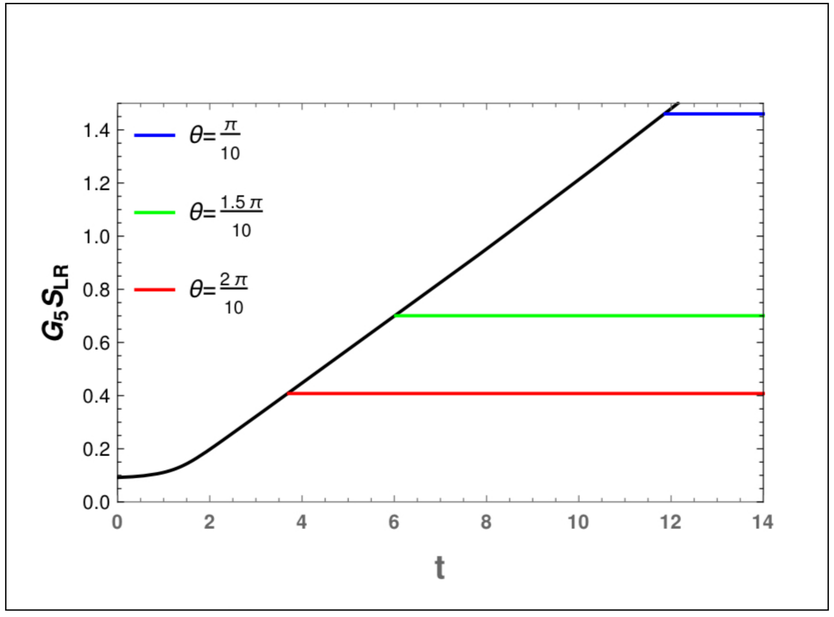



# A QUICK LOOK INTO ETERNAL BH PLOTS (DOUBLY HOLOGRAPHIC BRANEWORLD MODEL)

Based on 2103.15852 w/ A. Bhattacharyya, P. Nandy, & A. K. Patra JHEP 05 (2021)135


(Removed Region)
$\theta_1$
Defect
$\theta_2$
(Removed Region)
$\mu$
$u$
I
II
Bath Brane
KR Horizon
Island Surface
Physical Brane
Island
Island Solutions
Hartman-Maldacena
Island Surface
$y$
$z$


Defect
Right Brane
$\theta_1$
Left Brane
Island
surface
HM surface


$G_5 S_{LR}$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\theta=\frac{\pi}{10}$
$\theta=\frac{1.5\,\pi}{10}$
$\theta=\frac{2\,\pi}{10}$
0
2
4
6
8
10
12
14
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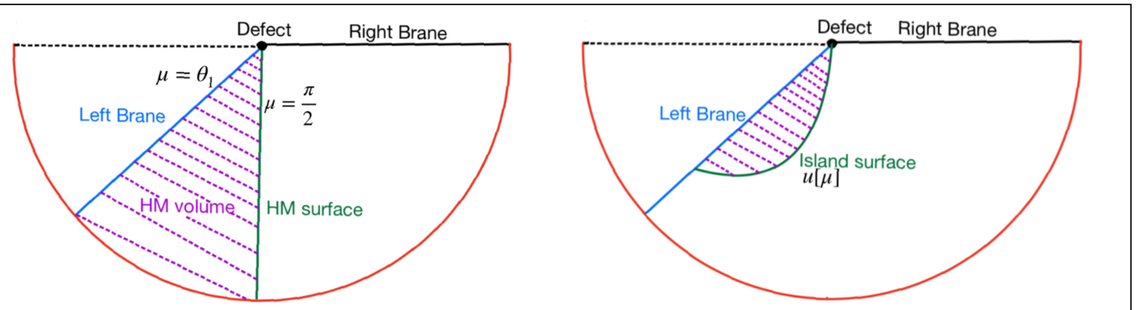
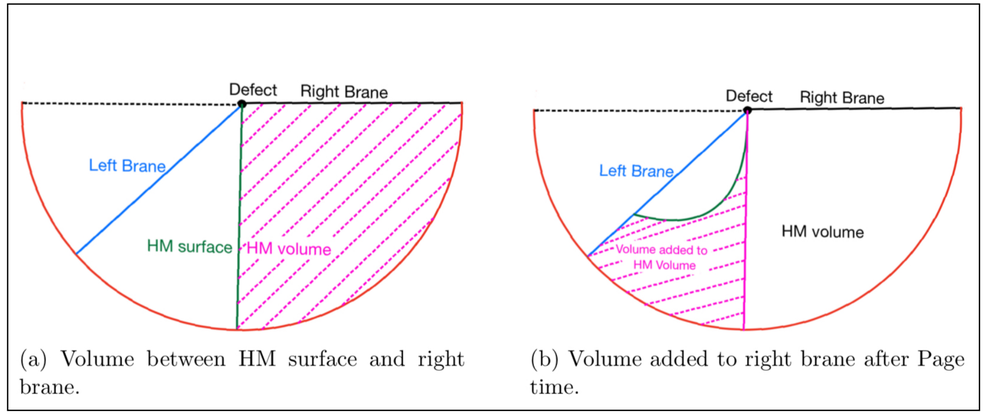


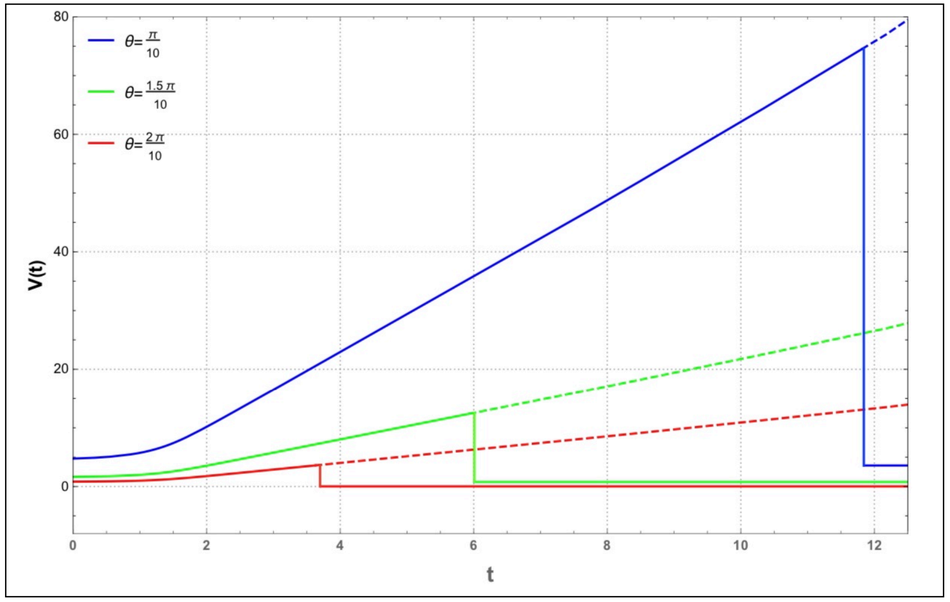

(a) Volume between physical brane and HM surface.

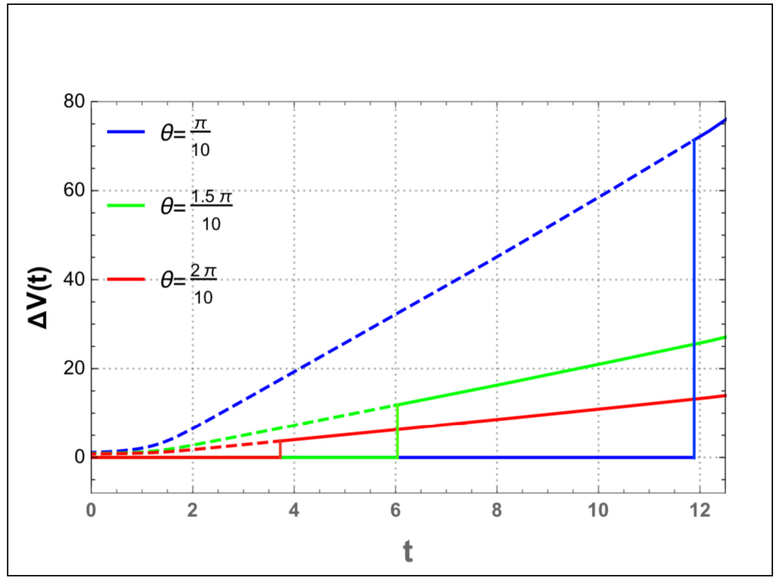
(b) Volume under island on physical brane.




(a) Volume between HM surface and right brane.

(b) Volume added to right brane after Page time.



# CONCLUSION AND FUTURE DIRECTIONS

- Inspite of the differences of the Page curves in the two models, the candidate complexity curves look similar.
- Finite jump in complexity at Page time →multipartite purification complexity (Caceres et al. & Maxfield)→ complexity of islands (Myers et al ' 2020)
- Island inclusion →Hawking modes purification.
- Looking for purification and QECC connections in quantum mechanical and $CFT_2$ systems. (Following works of (a)M. Flory & M. Heller on CFT complexity and (b)A. Dymarksky on QECC and CFT).

# THANK YOU